

## Engineering Economics:

### Recommended Books:

- E.P. Degrama, W.G Sullivan and J.A Bostadel
- N.N Borish and S. Kaplan “ Economic analysis for Engineering and Managerial Decision making.
- Gerald J. Thuesen, W.J Fabrycky
- Manuals (TU pulchok and kirah Thapa)

## Introduction:

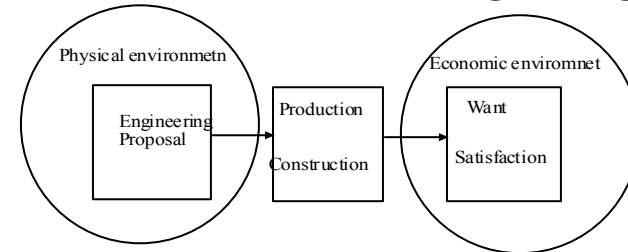
**Engineering:** Engineering is not a science but an application of science. It is an art composed of skill and ingenuity in adopting knowledge to the use of huminity. Engineering activities of analysis and design are not an end in themselves. They are means for satisfying human wants. Modern Civilization depends to a large degree upon engineering. Most products and services used to facilitate work, communication, transportation and national defence and to furnish sustenance , shelter and health are directly or indirectly a result of engineering activities.

**Economys:** Economy, the attainmet of an objective at low cost in terms of resource , input has always been associated with engineering.

Economics deals with the behavior of people individually and collectively, particularly as their behavior relates to the satisfaction of their wants.

Because of resource constants , engineering must be closely associated with economy. It is essential that engineering proposals be evaluated in terms of worth and cost before they are undertaken.

## The Bi-Environmental Natural of Engineering:



Engineering is concern with two interconnected environments the physical and economic.

Engineers are confronted with two environment, the physical and economic. Their success in altering the physical environment to produce products and services depends upon the knowledge of physical laws. However, the worth of this products and services lies in their utility measure in economic term. The usual function of engineering is to manipulate the elements of one environment, the physical , to create value in second environment, the economy.

Physical and Economical Efficiencies:

Physical effciecies:  $\text{Output/ Input.}$

= will always he less than unity or 100%

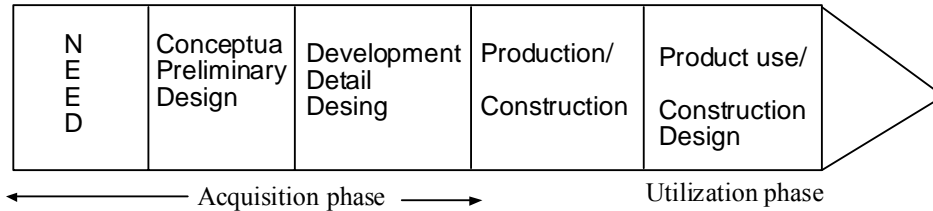
Economic efficiency:  $\text{worth/ cost}$

= economic efficiency can be exceeded 100%

In engineering activities, economic efficiencies must take precedence over physical efficiency. Thus is because the function

of engineering is to create utility in the economic environment by altering elements of the physical environment.

### Product life cycle:



In general engineers have focus mainly on the acquisition phase of the product life cycle and have been evolved in early design and analysis activities alone. Product performance has been main objective versus the development of an overall system with economic factors in mind. However, experience in recent decades indicates that a properly functioning product which is competitive in the market place cannot be achieved through efforts applied largely after the product comes into being

**Engineering Economy:** For an engineering design to be successful, it must be technically sound and produce benefits. The field of engineering economy is concerned with the systematic evaluation of the benefits and costs of projects involving engineering design and analysis. Thus, engineering economy requires the application of engineering design and analysis. Principles to provide goods and services that satisfy the consumer at an affordable cost.

Engineering economy involves the systematic evaluation of the economic merits of proposed solutions to engineering problems. To be economically acceptable

(affordable) solutions to engineering problems must demonstrate a positive balance of long term benefits over long term costs, and they must also

- Promote the well-being and survival of the organization.
- Embody creative and innovative technology.
- Permit identification and scrutiny of the estimated outcomes.
- Translate profitability

**Principles of Engineering Economy:** Once a problem/need has been clearly defined, the foundations of the discipline can be discussed in terms of seven principles.

1. Develop the alternatives
2. Focus on the differences
3. Use of a consistent viewpoint.
4. Use of a common unit of measure.
5. Consider all relevant criteria
6. Make uncertainty explicit
7. Revisit your decisions.

### Some business and Accounting Terminology:

**Value:** Value is measure of the worth that a person ascribes to a good or a service. Thus, the value of an object is inherent not in the object but in regard that a person has for it.

Value should not be confused with the cost or the price of an object. There may be little or no relation between the value a person ascribes to an article and the cost of providing it or the price that is asked for it.

**Utility:** It is a measure of the power of a good or a service to satisfy human wants. Thus the utility of an object like its value, inheres not in the object itself but in the regard that a person has

for it. Utility and value are closely related. The utility that an object has for a person is the satisfaction he or she derives from it.

**Cost/Price:** The term cost and price are often use together. The cost of a product or service is the total of resources , direct and indirect , required to produce it. The price is the value of good or service in the market place. In general price is equal to cost plus profit. Two class of goods are recognized by economist: consumer goods and producer goods . Consumer goods are the goods and services that directly satisfy human wants. Examples are consumer goods are T.V sets, house, shoes, book. Producer goods are the goods or services the indirect as part of production or construction process. Example: bulldozer, machine tools, raw material.

**Interest:** It is rental amount charged by financial institutions for the use of money.

**Interest Rate or the rate of capital growth:** It is the rate of gain received from an investment.

**Worth:** Worth is the equivalent of money in a commodity(good or service).

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**Annuity:** An amount of money payable to a beneficiary at regular interval for a prescribed period of time out of a fund reserved for that purpose.

- A series of equal payments occurring at regular periods of time.

Figure:

**Assets:** Capital owned by a company (cash, receivable, equipment building, land etc.)

**Liability:** liabilities are those things of monetary values that the company owes. (Short term dept, long debt, payables)

**Equity:** Equity is the worth of what the company owes to its stock holders ( capital stocks)

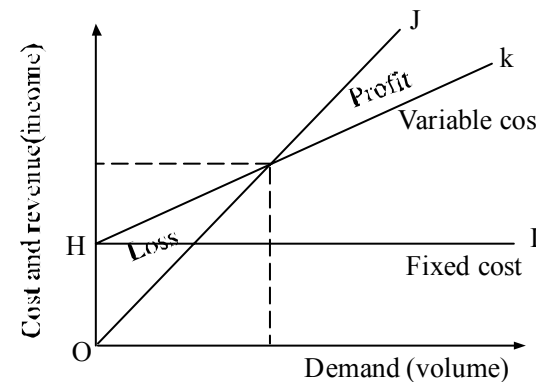
**Fundamental accounting equation:**

Assets = liability + owner's equating.

Revenues – expenses = profit (or loss)

Break Even point:

1. In business Operations, the rate of operations, outputs or sales at which income is sufficient to equal operating cost .
2. The % of capacity operation of manufacturing plant at which income will just cover expenses.



HL = Fixed Production cost

HK = Fixed cost +variable cost

OJ = Income from sell

**Capital:** The non-human ingredients that contribute to the production of goods and services including land, raw, semi finished materials, tools, buildings, machinery and inventories.

**Cash:** The actual rupees coming into a firm (cash inflow) or paid out by a firm (cash out flow).

**Compound interest:** (a) The type of interest that is periodically added to the amount of investment (or loan) so that subsequent interest is based on the cumulative amounts.

(b) The interest charges under the condition that interest is charged on any previous interest earned in any time period as well as the principal.

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**Decision Making:** A program of action undertaken as a result of established policy to influence the final decision.

**Decision under certainty:** Simple decision that assume complete information and the uncertainly connected with the analysis of the decisions.

**Decision Under taken:** A decision problem in which the analyst elects to consider several possible futures, the probabilities of which can be estimated .

**Decision under Uncertainty:** A decision for which the analyst elects to consider several possible futures, the probabilities of which can not be estimated.

**Demand:** The various quantities per unit of time of an item that a buyer is willing to buy at all alternative price other things being equal.

**Discount Rate:** The rate used to calculate the present value of series of future cash flows. Inverse of compounding.

**Depreciation:** (a) Decline in value of a capitalized cost.  
(b) Loss of value because of obsolescence or due to attrition.

**Economic Goods:** Anything that is useful, transforable and not abundant.

**Economic life:** The length of time an asset will be economically useful.

**Economic efficiency:** The ratio of the value of outputs obtained from an economic process to the value of inputs necessary to produce them. The higher the value of output per rupees worth of resource input, the greater the efficiency of the process.

**Intangibles:** (a) Conditions or economic factor's that cannot be readily evaluated in quantitative terms as in money.

(b) The assets that can not be reliably evaluated.

**Inflation:** A condition is which the price level increases rapidly.

**Labour:** The capacity of human effort (both mind and muscle) available for use in producing goods and services ranging from unskilled, skilled, specialized labor power.

**Opportunity cost:** The cost associated with an alternative or foregone opportunity that a firm or individual bypasses.

**Payback period:** Regarding an investment, the number of years( or months) required for the related profit or saving in operating cost to equal the amount of said investment.

**Simple Interest :** The interest changes under the condition that interest at any time is charged only on the principal.

**Study Period:** In economic study, the length of time that is presumed to be covered in the schedule of events and appraisal of results. Often the anticipated life of the project under

consideration, but a shorter time may be more appropriate for decision making.

**Supply:** The various quantities per unit of time of an item that a seller is willing to sell at all alternative prices.

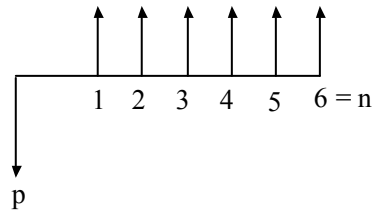
**Time value of money:** The expected interest rate that capital should or will earn.

## 1.2 Cash flow:

The actual rupees coming into or out of a firm.

Cash inflow: Actual rupees coming into a firm.

Cash outflow: Actual rupees paid out by a firm.



The use of cash flow (time) diagrams/ tables is strongly recommended for situations in which the analyst needs to clarify or visualize what is involved when flows of money occurs at various times.

The cash flows employs several conditions:

1. The horizontal line is time scale with progression of time moving from left to right. The period (eg year, quarter, month) labels can be applied to intervals of time rather than to point on the scale.
2. The arrows signify cash flows and are placed at the end of the period. If a distinction needs to be made, downward arrows represent expenses ( negative cash flow or cash outflows) and upward arrows represents receipts (positive cash flow or cash inflow).

3. The Cash flow diagram is dependent on the point of views. For examples the cash flows shows above in the figure are based on the cash flow as seen by the lender.

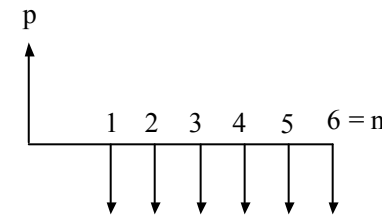


Figure: Borrower's view point.

**1.3 Economic system:** Economics system is the institutional frame work within which a society or country carries on its economic activities.

On one extremity we have private enterprises system which is characterized by private ownership of resources as well as goods and services. Private individuals, business enterprises and association of various kinds can engaged in whatever voluntary production and exchange activities they desire.

On the other extremity, we have pure socialistic system where there is no private property. Resources, goods and services are owned or controlled by the government. Production takes place in government enterprises and the government specifies the condition under which exchange can offer. The private enterprise system is decentralize where as socialistic is highly centralize. Present day economics are a mixture of socialism and private enterprises. Some part of an economics output will be produce by the profit oriented private sector of the economy. Another part will be produce in an socialist manner by the public sector. There are also non profit sectors like hospital, school.

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**2 .Cost classification and analysis:** In engineering economics cost classification and analysis provide vital information. Which is subsequently used for proper planning, decision making and control of a project.

**2.1 Element of cost:** The element of cost are material cost, labour cost , expenses.

**Material cost:** These are the costs of raw materials, spare parts , consumable material, packing materials.

**Labour cost:** Costs of unskilled as well as skilled labour employed in construction and production process fall into this category.

**Expenses:** These include costs of special designs, drawings or layouts, cost of purchase or hire of tools and plants (equipments) for a particular job and maintenance cost of such tools and equipment.

2.2 Classification of cost: primary cost and overhead cost

The element of cost mention above, i.e material cost, labour cost and expense, constitute the total cost. The total is cost is generally calssified into prime cost and overhad cost.

**Prime cost:** It is the direct cost that can be reasonably measure and allocated to a specific output or work activity. Thus it is the aggregate of direct material cost, direct labour cost and direct expenses.

**Overhead cost:** It is indirect cost that cannot be reasonably measure and allocated to specific output or activities. Overhead

cost is the sum of indirect material cost, indirect labour cost, indirect expenses.

**Component of prime cost:**

**a. Direct material cost:**

- i. Material including component parts, specially purchased requisitioned for a particular job ordered or processed.
- ii. Material processing from one operation or process to another.
- iii. Primary packing materials such as cartoon cardboard box etc.

**b. Direct labour cost:**

i. Labour engaged in altering the condition , conformation and composition of the product.

ii. Inspector, analyst etc specially required for such production.

iii. If specially identified, the wage of foreman, chargehands, shop-clerks, ways of internal transport personal etc.

**c. Direct expenses:**

- i. cost of special design, drawing and layouts.
- ii. Hire of special tools and equipment for a particular job.
- iii. Maintenance cost of such tools and equipment.

**Components cost of Overhead cost:** Overhead cost may be of 4 types – production overhead, administration overhead, selling overhead, distribution overhead.

Here component of production are given:

- i. Indirect material cost.
- ii. Indirect labour cost.
- iii. Indirect expenses.

**Indirect material cost:** Indirect material is the material that can not be traced in the finished product. E.g lubricants , cotton, waste, grease, small tools. How ever some minor items which enter into production and form pars of its are conveniently treated as indirect material. Such as cost of thread in shirt in shirt making cost.

**Indirect labour cost:** Labour cost not charged directly are indirect labour cost. In general salary or wage of the following are treated as indirect wages: foreman, supervisor, maintenance labour etc.

**Indirect expenses:**

- a. Rent, rates(taxes) , insurance in relation to factory.
- b. Depression, power and fuel, repair and maintenance of plant machinery and building.
- c. Other sundry expenses like first aid employment etc.

*Date: 2065/4/27*

*Q. Following are the data for the production of a 100 badminton racquets:*

*Labour rate: Rs. 40/hr*

*Leather: 50m at Rs. 200/metre*

*Gut: 300 m at Rs. 50 /meter.*

*Graphite: 100kg at Rs. 200/kg*

*Total annul factory overhead : Rs. 500000*

*Total annual direct labor hours: 25000 hrs*

*Total annual direct Labour hours: 25000 hrs*

*Labour hours needed: 200 hrs.*

*Show the cost breakdown and calculate th total cost for per racquet.*

Solution:

Total annual direct Labour hours = 250000 hrs.

Labour hours needed for 100 racquets = 200 hrs

Therefore, Total production capacity  
per year =  $(25000/200)*100$   
= 125000 Nos.

**Cost breakdown for 125000 Nos. racquet.**

Prime costs:

(a) Labour cost:  $40* 200/100*125000 = \text{Rs. } 10000000$

(b) Material cost:

i. Cost of leather =  $((50/100)*200*125000)/100$

ii. Cost of Gut =  $(300/100)*50*125000 = \text{Rs. } 18750000$

iii. Cost of Graphite =  $(100/100)*200)125000 = \text{Rs. } 25000000$

Therefore , total direct material cost = Rs. 56250000

Therefore Total cost for production of 125000 badminton racquet = Labour cost + material cost.

= Rs. 10000000+56250000

= Rs. 66250000

Overhead cost = Rs. 5000000

Therefore, cost for production of 125000 No. of badminton for racquets

= prime cost +overhead cost

= Rs. 66250000+ Rs. 5000000

= Rs. 71250000

Therefore,

Manufacturing cost of each racquet = Rs. 71250000/125000

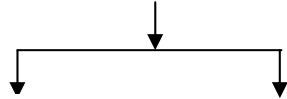
= Rs. 570

**2.3 Cost variance analysis:** When actual performance are recorded and compared with the set of standard some deviation are observed. These deviations are called variances. Variance may favorable or adverse. If

Actual cost < standard cost = favorable

Actual cost > standard cost = adverse

**Direct materials cost variance:** It is difference between standard cost of direct materials specified for output achieved and actual cost of direct material used.



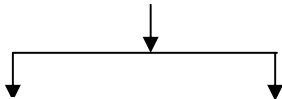
I) Direct material Price cost

ii) Direct material uses variance.

i) Direct material price variance – (Actual quantity consumed X actual rate) – (actual quantity consumed x standard rate)  
= AQ(AR-SR)

ii) Direct material usage variance: = ( Actual quantity consumed x st. rate) –(st. quantity specified x standard rate)  
= .....

**Direct wage variance:** This is the variance between standard direct wages specified for the activity achieved and actual direct wage paid.



I) Direct wage rate variance

ii) Direct labour efficiency variance

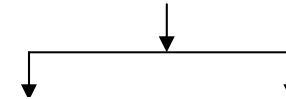
Direct wage rate variance:

- Actual hours worked at actual rate- Actual hours worked at standard rate.
- AH(AR-SR)

Direct labour efficiency variance:

- Actual hours worked at stand rate- standard hours worked at standard rate.
- AH\*SR-SH\*SR
- SR(AH-SH)

**c) Variable overhead variance**



I) Variable overhead Expenditure variance

ii) Variable overhead efficiency variance.

Variable overhead expenditure variance:

$$= AH * AR - AH*AR$$

$$= AH(AR-SR)$$

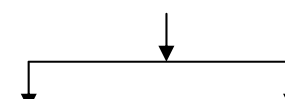
Variable overhead efficiency variance:

$$= AH*SR - SH*SR$$

$$= SR(AH- SH)$$

**d) Fixed overhead cost:**

Actual fixed overhead cost- standard fixed overhead cost.



I) Fixed overhead expenditure variance

Fixed overhead efficiency variance.



$$(i) \quad \text{Direct material price variance} = AQ (AR - SR) \\ = 56250(24 - 25) \\ = 56250 \times -1 \\ = -Rs. 56250 \text{ (Therefore variance)}$$

**Fixed overhead expenditure:**

$$\text{Actual fixed overhead} - \text{budgeted fixed overhead} \\ = AH \times AR - AH \times SR \\ = AH(AR - SR)$$

**Fixed overhead efficiency variance:**

$$AH \times SR - SH \times SR \\ = SR(AH - SH)$$

$$(ii) \quad \text{Direct material uses variance:} = SR(AQ - SQ) \\ = 25(56250 - 50000) \\ = Rs. 156250 \text{ (Therefore adverse variance)}$$

Therefore, Total material cost variance = Direct material price variance + Direct material cost variance.

Date: 2065/4/29

Q. The information given below shows the records of a manufacturing company comparing the actual data with the data from standard cost card.

	Standard	Actual
Production (units)	10000	7500
Direct material (kg)	50000	56250
Direct material cost (Rs.)	1250000	1350000
Direct labour cost (Rs)	11500000	7,425,000
Fixed overhead cost (Rs)	13,800,000	7,796,250
Variable overhead cost (Rs)	9,200,000	5,197,500

- Calculate:
- total material cost variance.
  - Total wage variance
  - Variable overhead variance
  - Fixed overhead variance.

Indicating the separate components of each variance. Also indicate favorable or adverse variance

a) Total material cost variance:

$$= -Rs 56250 + Rs. 156250 \\ = Rs. 10000$$

Hence, Adverse variance.

(c) Total wage variance:

$$(i) \quad \text{direct wage rate variance} = AH(AR - SR) \\ = 67500(10 - 115) \\ = -Rs. 337500 \text{ (favorable variance)}$$

$$(ii) \quad \text{Direct labour efficiency variance} = (AH \times SR) - (SH \times SR) \\ = (AH \times SR) - ((AP \times SH / \text{unit}) \times SR) \\ = 67500 \times 115 - ((7500 \times 10,000 / 10,000) \times 115) \\ = 7762500 - 8625000 \\ = -Rs. 862500 \text{ (favorable variance)}$$

$$\text{Therefore, Total usage variance} = (i) + (ii) \\ = -Rs. 337500 + Rs(-862500) \\ = -Rs. 120000$$

Hence the variance is favourable.

(c) Variable overhead variance:

[See in manual for theory]

(i) Variance overhead Expenditure variance

$$\begin{aligned}
 &= AH * AR - AH*SR \\
 &= AH(AR-SR) \\
 &= 67500( (5197500/67500) - (9700000/100000)) \\
 &= 67500( 77- 92) \\
 &= -Rs. 1012500( favourable variance)
 \end{aligned}$$

(iii) Variable overhead efficiency variance:

$$\begin{aligned}
 &= (AH*SR) - (AP*SH/unit*SR) \\
 &= ( 67500*92)- (7500*100000/10000*92) \\
 &= -Rs. 69000 (favaurable variance)
 \end{aligned}$$

Therefore , Variance overhead variance = (i) +(ii)

$$\begin{aligned}
 &= -Rs.1012500+ (-Rs.690000) \\
 &= -Rs. 1702500
 \end{aligned}$$

Hence Favourable variance.

(d) Fixed overhead variance:

(i) Fixed overhead expenditure variance:

$$\begin{aligned}
 &= AH( AR-SR) \\
 &= 67500( 7796250/67500 - 13800000/100000) \\
 &= 67500( 115.50-138) \\
 &= -Rs. 1518750 (favaourable )
 \end{aligned}$$

(ii) Fixed overhead efficiency variance.

$$\begin{aligned}
 &= (AH* SR) - ( AP*SH/unit* SR) \\
 &= ( 67500-*138) - ( 7500*100000/10000*138) \\
 &=-Rs. 1035000 ( favourable)
 \end{aligned}$$

Therefore , Total fixed overhead cost = (i) + (ii)

$$\begin{aligned}
 &= -Rs 1518750+(-Rs. 1035000) \\
 &= -Rs. 2553750
 \end{aligned}$$

(hence favourable variance)

## 2.4 Job Costing and Process costing:

*Date: 2065/4/31*

# Product A is produced after three distinct processes. The following information is obtained from the accounts of a period.

Items	Total	Process		
		I	II	III
Direct material	3000	2000	500	500
Direct wages	500	100	300	100
Direct expenses	600	300	100	200

The production overhead is Rs.1000 and is 200% of direct wages . Production was 100 kg. No opening or closing stocks prepare process cost accounts assuming no process loss,

Solution:

Items	Process cost account			
	Process-I	Process-II	Process-III	Tot.
i. direct material	2000	500	500	3000
ii. direct wages	100	300	100	500
iii. direct expenses	300	100	200	600
iv. pro. Overhead	200	600	200	1000
Total cost for each:	2600	1500	1000	5100
Process cost for each:	26	15	10	51

Process

Now , production was 100 kg. No opening or closing stocks, Hence ,

The cost of product A is Rs. 5100 and production per unit (kg). is = Rs. 5100/100Kg. = Rs. 51 per kg.

## 3.0 Interest and time value of money:

Because of money can earned at a certain interest rate through its investment for a period of time, a rupee received at some future date is not worth as much as a rupee in hand at present . This relationship between interest and time leads to the concept of time value of money.

A rupee in hand now is worth more than a rupee received after ‘n’ year from now . Why? Because having the rupee now provides the opportunity for investing that rupee ‘n’ years more than a rupee to be received in ‘n’ years. Since money has earning power, this opportunity will earn a return, so that after ‘n’ years the original rupee + its interest will be large amount than the rupee received at that time.

Year(n)	Amount at Beginning of Year(P)	Compound amount At end of year (A) =(P+i)	Interest=Pni/100 i = R/100
1	P	P(1+i)	Pi
2	P(1+i)	P(1+i) <sup>2</sup>	P(1+i)i
3	P(1+i) <sup>2</sup>	P(1+i) <sup>3</sup>	P(1+i) <sup>2</sup> i
.	.	.	.
.	.	.	.
.	.	.	.
n-1	P(1+i) <sup>n-2</sup>	P(1+i) <sup>n-1</sup>	P(1+i) <sup>n-2</sup> i
n	P(1+i) <sup>n-1</sup>	P(1+i) <sup>n</sup>	P(1+i) <sup>n-1</sup> i

**3.1 Simple interest; compound interest, interest tables, interest charts.**

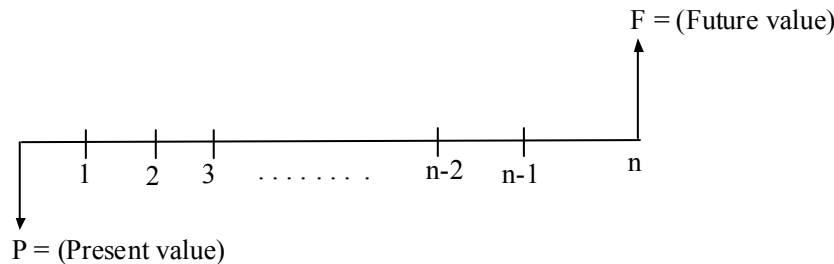
1. Simple interest:  $I = PTR/100$

2. Compound interest:

**Discrete compounding and discrete cash flow:**

a. Single payment (single cash flow):

Compound amount factor (F/p, i, N) [i = R/100]



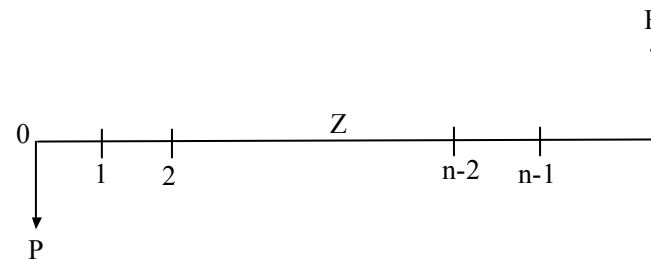
Therefore,  $A_F = P(1+i)^n$

In functional form , this is written as

$$F = P(F/P, i, n)$$

The factor  $(1+i)^n$  is known as single payment compound amount factor.

**b. Single payment present worth factor (p/F, i, n)**



We know,

$$F = p(1+i)^n$$

Therefore,  $P = F/(1+i)^n = F(1+i)^{-n}$

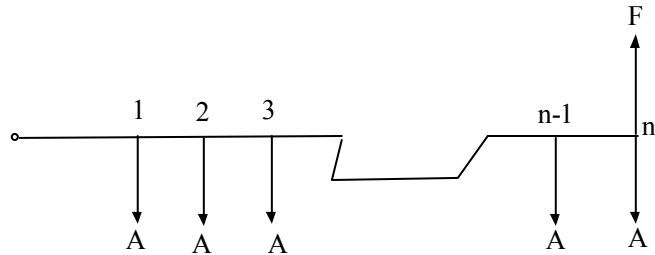
In functional form,

$$P = F ( p/F, i, n )$$

The factor  $(1+i)^{-n}$  is known as single payment present worth factor and is designated as  $(p/F, i, n)$

**C. Equal-payment, series compound amount factor (F/n, i, n)**

Cash flow



Equal annual series and future amount.

$$F = A + A(1+i)^1 + A(1+i)^2 + \dots + A(1+i)^{n-1} \dots (i)$$

$$F(1+i) = A(1+i) + A(1+i)^2 + \dots + A(1+i)^n \dots (ii)$$

From (i) and (ii)

$$\begin{aligned} F_i &= A(1+i)^n - A \\ F &= A \{ (1+i)^n - 1 \} \\ &= A ( F/A, i\%, n ) \end{aligned}$$

This can be written in the functional form as

$$F = A ( F/A, i, n )$$

The factor  $\left( \frac{(1+i)^n - 1}{i} \right)$  is known as equal payment series compound amount factor.

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**d. Equal – payment series sinking fund factor [ A/F, i, n]**

$$A = F \cdot \left( \frac{i}{(1+i)^n - 1} \right)$$

In general form;

$$A = F ( A/F, i, n )$$

The resulting factor  $\left( \frac{i}{(1+i)^n - 1} \right)$  is known as payments series sinking fund factor and is designated  $[A/F, i, n]$

**e. Equal payment series capital Recovery Factor (A/F, i, n)**

Figure:

We know that  $A = F \cdot \left( \frac{i}{(1+i)^n - 1} \right) \dots (i)$

And  $F = P(1+i)^n \dots (ii)$

Putting the value of F in equation (i)

$$A = P(1+i)^n \left( \frac{i}{(1+i)^n - 1} \right)$$

$$A = p \left( \frac{i(1+i)^n}{(1+i)^n - 1} \right)$$

The resulting factor  $\left( \frac{i(1+i)^n}{(1+i)^n - 1} \right)$  is known as equal payment series capital Recovery factor.

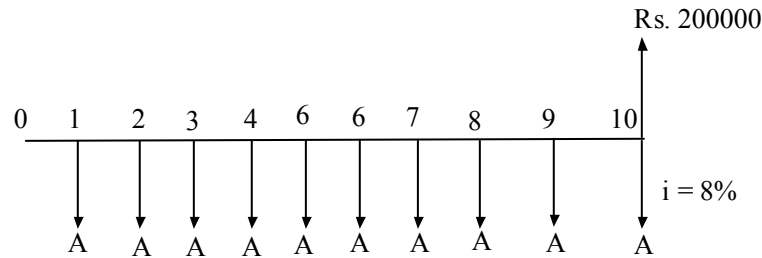
**f. Equal payment series Present worth factor ( P/A, i, n)**

Therefore  $P = A \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right)$

The resulting factor  $\left( \frac{(1+i)^n - 1}{i(1+i)^n} \right)$  is known as equal series present worth factor and is denoted by  $(P/A, i, n)$

# (1) A man wants to have Rs 200000 for the studies of his son after a period of 10 yrs. How much does he have to deposit each year in a saving account that earns 8% per year?

Solution:



Given No of years  $(n) = 10$  yrs  
 Interest rate,  $i = 8\% = 0.08$   
 Future worth,  $F = \text{Rs. } 200000$   
 Annuity,  $A = ?$

We know that,  $A = F \left( \frac{i}{(1+i)^n - 1} \right)$

Sinking Factor =  $\left( \frac{i}{(1+i)^n - 1} \right) = 0.08 / (1 + 0.08)^{10} - 1 = 0.069$

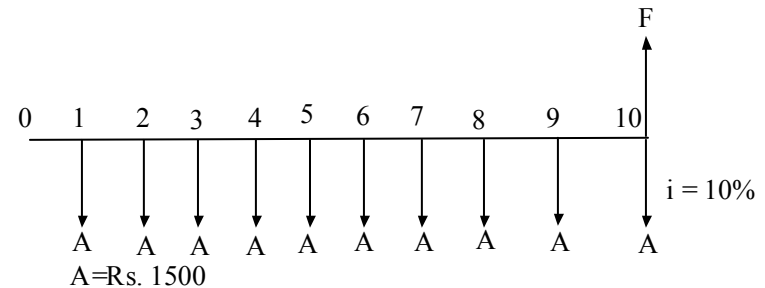
Therefore,  $A = \text{Rs } 200000 \times 0.069 = \text{Rs. } 13800$

Hence, the man has to deposit Rs 13800 each year in the saving account

# (2) If a woman deposits Rs. 1500 at the end of each year for 10 continuous years, how much money is accumulated at the end of 10 years?  $i = 10\%$  compounded annually.

Solution:

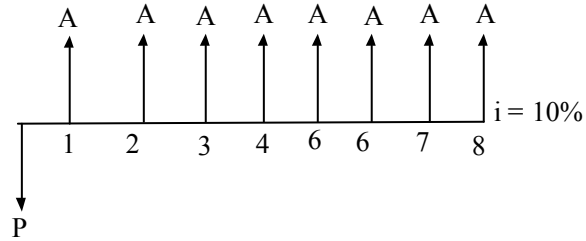
Given, No. of year,  $(n) = 10$  yrs  
 Interest rate,  $i = 10\% = 0.1$   
 Annuity,  $A = \text{Rs. } 1500$



We know that,  $F = A \cdot \left( \frac{(1+i)^n - 1}{i} \right)$

Equal payment series compound amount factor =  $\left( \frac{(1+i)^n - 1}{i} \right)$   
 $= (1+0.10)^{10} - 1 / 0.10 = 15.937$

#(3) How much money should you deposit, now in a saving account earning 10% interest rate compounded annually so that you may make eight end of year withdrawals of Rs. 2000 each.



$i = 10\%$   
 $n = 8$  years  
 $A = \text{Rs. } 2000$   
 $P = ?$

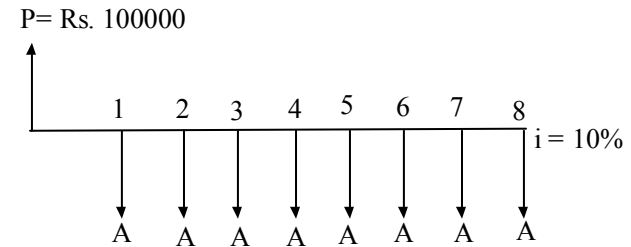
We know that,  $P = A \cdot \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right)$

Present worth factor =  $\left( \frac{(1+i)^n - 1}{i(1+i)^n} \right)$  or,  $(p/A, i, n)$   
 $= (1 + 0.10)^8 - 1 / 0.10(1 + 0.10)^8$   
 $= 5.3349$

$P = 2000 \times 5.3349$   
 $= 10669.8$

#(4) If a man wishes to repay a loan of Rs. 100000 by paying equal amount at the end of 8 successive years, what is the end of year payment? The first payment is due one year after receiving the loan,  $i = 10\%$ .

Solution:



$i = 10\% = 0.10$   
 $n = 8$  yrs  
 $P = \text{Rs. } 100000$   
 $A = ?$

We know that

$$A = P \left( \frac{i(1+i)^n}{(1+i)^n - 1} \right)$$

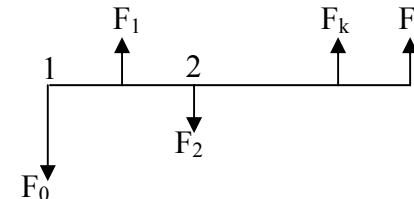
$$= 100000(0.10(1+0.10)^8) / ((1+0.10)^8 - 1)$$

$$= 100000 \times 0.1875$$

$$= 18750$$

**3.2 Present Worth (PW):** Present worth (PW) is the equivalent worth of all cash flows relative to some base point in time is called present, i.e all cash in flows and outflows discounted to a base point at interest MARR ( minimum Attractive Rate of Return). Discounting future amounts to present,

**Figure:**



$$PW = F_0 + F_1(1+i)^{-1} + \dots + F_k(1+i)^{-k} + \dots + F_n(1+i)^{-n}$$

14 Where,  $i =$  effective interest rate or MARR

n = number of compounding year.  
F<sub>k</sub> = Future cash flow at the end of period k.

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### 3.3 Nominal and effective Interest rates:

The basic annual rate of interest is known as nominal interest rate denoted by 'r'.

The actual or exact rate of interest earned on the principal during one year is known as effective interest rate denoted by 'i'.

Relationship between nominal interest and effective interest rate is given by :

$$i = \left(1 + \frac{r}{m}\right)^M - 1$$

Where M is the no of compounding period per year.

# Nominal rate of 12% compounded monthly with time interval of one year. [ M = 12].

$$i = \left(1 + \frac{r}{m}\right)^M - 1$$

$$i = \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

= 0.1268 or, 12.68% per year.

# Nominal rate of 18% compounded weekly with the time interval of one year [ M = 52]

$$i = \left(1 + \frac{r}{m}\right)^M - 1$$

$$i = \left(1 + \frac{0.18}{52}\right)^{52} - 1$$

# Nominal rate 14% compounded monthly with a time interval of six months.

$$i = \left(1 + \frac{r}{m}\right)^M - 1$$

$$i = \left(1 + \frac{0.14}{12}\right)^6 - 1 = 0.0721 \text{ or, } 7.21\% \text{ per six months.}$$

# Nominal rate of 10% compounded weekly with a time interval of six months.

$$i = \left(1 + \frac{r}{m}\right)^M - 1$$

$$i = \left(1 + \frac{0.10}{52}\right)^{26} - 1 = 0.0519 \text{ or } 5.19\% \text{ for 6 months.}$$

#Nominal rate of 13% compounded monthly with a time interval of two years.

$$i = \left(1 + \frac{r}{m}\right)^M - 1$$

$$i = \left(1 + \frac{0.13}{12}\right)^{24} - 1 = 0.2951 \text{ or, } 29.51\% \text{ per two years.}$$

### 3.4 Continuous Compounding and continuous compounding formula:

Continuous compounding assumes that cash flows occurs at discrete intervals but the compounding is continuous throughout the interval.

(a) Single – payment compound amount factor:

For annual compounding  $F = P(1+r)^n$

For semi-annual compounding  $F = P(1+r/2)^{2n}$

For monthly compounding  $F = P(1+r/12)^{12n}$

In general, if there are m compounding periods per year.

$$F = P(1+r/m)^{mn}$$

$$e^r = (1+i)$$

$$i = e^r - 1$$

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### 3.4 Continuous Compounding and continuous compounding formula:

Continuous compounding assumes that cash flows occur at discrete intervals but the compounding is continuous throughout the intervals.

- a) Single –payment compound amount factor for annual compounding.

$$F = P(1+r)^n$$

For semi-annual compounding

$$F = p(1+r/2)^{2n}$$

For monthly compounding

$$F = p(1 + r/12)^{12n}$$

In general , if there are m compounding periods per year.

$$F = p(1+r/m)^{mn}$$

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When interest is assumed to compound continuously. The interest earned is instantaneously added to the principal at the end of each infinitesimal interest period. For continuous compounding , the number of compounding periods pre year is considered to be infinite.

$$\text{Therefore, } F = p * [\lim_{m \rightarrow \infty} (1+r/m)^{mn}]$$

$$= p * [\lim_{m \rightarrow \infty} \{ (1+r/m)^{m/r} \}^{r.n}]$$

$$\text{But, } \lim_{m \rightarrow \infty} (1+r/m)^{m/r} = e = 2.7182$$

$$F = pe^{rn}$$

For n years the compound amount factor will be  $e^{r.n}$ . Since  $e^{r.n}$  for continuous compounding corresponding to  $(1+i)^n$  for discrete compounding.

- b) Single payment present worth factor ( P/F, r, n)

$$\text{We know } F = pe^{r.n}$$

$$\text{Therefore, } P = F \cdot [ 1/e^{r.n}]$$

The resulting factor  $e^{-r.n}$  is the single payment present worth factor for continuous compounding interest and is designated as (P/F, r, n)

- c) Equal payment series compound amount factor (F/A, r, n):

We know for discrete compounding  $F = A[((1+i)^n - 1)/i]$  and

$$e^r = 1+i$$

$$i = e^r - 1$$

$$\text{Therefore, } F = A [ (e^{r.n} - 1)/e^{r.n}]$$

The resulting factor  $(e^{r.n} - 1)/e^{r.n}$  is the equal payment series compound amount factor for continuous compounding interest and is designated as [F/A, r, n]

- d) Equal payment series sinking fund factor.( A/F, r, n)

$$A = F[(e^r - 1)/e^{r.n} - 1]$$

The resulting factor  $[(e^r - 1)/e^{r.n} - 1]$  is the equal payment series sinking fund factor for continuous compounding interest and is designated as (A/F, r, n)

- e) Equal payment series present worth factor (P/A, r, n)

$$\text{We know for discrete compounding } P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\text{As we have, } e^r = 1+i, \quad i = e^r - 1$$

$$\text{For continuous compounding } P = A \left[ \frac{e^{r.n} - 1}{(e^r - 1)e^{r.n}} \right]$$



The resulting factor  $\left[ \frac{e^{r.n} - 1}{(e^r - 1)e^{r.n}} \right]$  is the equal payment series present worth factor for continuous compounding and is designated as  $[P/A, r, n]$ .

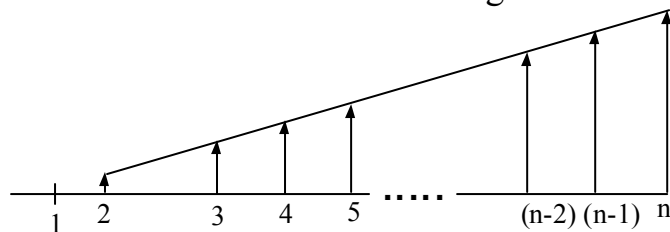
f) Equal payment series capital recovery factor  $(A/P, r, n)$

$$A = P \left[ \frac{(e^r - 1)e^{r.n}}{e^{r.n} - 1} \right]$$

The resulting factor  $\left[ \frac{(e^r - 1)e^{r.n}}{e^{r.n} - 1} \right]$  is the equal payment series capital recovery factor for continuous compounding and designated as  $[A/P, r, n]$

### 3.5 Interest calculation for uniform gradient:

Some time we have to deal with receipts or expenses that are projected to increase or decrease by a uniform amount, each period, thus constituting an arithmetic sequence of cash flows. For example, maintenance and repair expenses on specific equipment may increase by a relatively constant amount each period. It can be shown in the form of uniform rate 'n' given below.



The cash flow diagram given above shows a sequence of end of period cash flow increasing by a constant amount  $G$ , each period.  $G$  is known as uniform gradient amount. It should be noted that the cash flows start from the end of 2<sup>nd</sup> period onwards. All the formulae given here are based on this assumption

End of year	Gradient series	Set of series Equivalent to Gradient series	Annual series
0	0	0	0
1	0	0	0
2	G	G	A
3	2G	G+G	A
4	3G	G+G+G	A
.	.	.	.
.	.	.	.
.	.	.	.
(n-1)	(n-2)G	G+G.....+G	A
n	(n-1)G	G+G+G.....+G	A

Finding F when G is given.

$$F = G ( F/A, i, n-1) + G(F/A, i, n-2)+.....+G(F/A, i, 2)+G(F/A, i, 1)$$

$$= G \left[ \frac{(1+i)^{n-1} - 1}{i} \right] + G \left[ \frac{(1+i)^{n-2} - 1}{i} \right] + ..... + G \left[ \frac{(1+i)^2 - 1}{i} \right] + G \left[ \frac{(1+i)^1 - 1}{i} \right]$$

$$= \frac{G}{i} [(1+i)^{n-1} + (1+i)^{n-2} + ..... + (1+i)^2 + (1+i)^1 - (n-i)]$$

$$= \frac{G}{i} [(1+i)^{n-1} + (1+i)^{n-2} + ..... + (1+i)^2 + (1+i)^1 + 1] - \frac{nG}{i}$$

The bracketed term constitute the equal payment series compound amount factor for n years.

$$\frac{G}{i} \left[ \frac{(1+i)^n - 1}{i} \right] - \frac{nG}{i}$$

Fin A when G is given:

$$\begin{aligned} \text{We know that } A &= F \left[ \frac{1}{(1+i)^n - 1} \right] \\ &= \frac{G}{i} \left[ \frac{(1+i)^n - 1}{i} \right] \left[ \frac{1}{(1+i)^n - 1} \right] - \frac{nG}{i} \left[ \frac{i}{(1+i)^2 + 1} \right] \\ &= \frac{G}{i} - \frac{nG}{i} \left[ \frac{i}{(1+i)^{n^2} - 1} \right] \end{aligned}$$

Therefore,  $A = G \left[ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]$  is called unipolar gradient series

factor and is designed  $G (A/G, i, n)$

Finding  $p$  when  $G$  is given,

$$\text{As } P = \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\begin{aligned} \text{Or, } P &= G \left[ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right] \left[ \frac{(1+i)^n - 1}{1(1+i)^n} \right] \\ &= G \left[ \frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n} \right] \end{aligned}$$

$$P = G \left[ \frac{(1+i)^n - 1 - ni}{i^2(1+i)^n} \right]$$

The term inside bracket is called present worth conversion factor.

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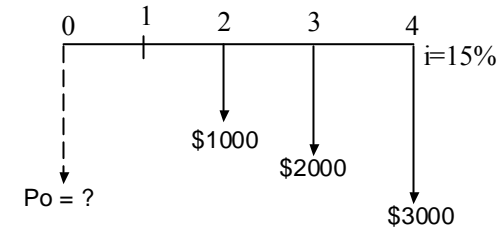
# Suppose that certain of year cash flow are expected to be \$1000 for the second year, \$2000 for the third year and \$3000 for fourth year and interest is 15% per year, it is designed to find the (a)

present equivalent value of first year (b) uniform annual equivalent value at the end of each of the 4 year.

$$G = \$1000$$

$$n = 4$$

$$i = 15\%$$



(a) The present equivalent can be calculated as.

$$\begin{aligned} P_0 &= G (p/G, 15\%, 4) \\ &= \$1000 \left[ \frac{(1+i)^n - 1 - ni}{i^2(1+i)^n} \right] \\ &= \$1000 \left[ \frac{(1+0.15)^4 - 1 - 4 \times 0.15}{0.15^2(1+0.15)^4} \right] \\ &= \$1000 * (3.79) \\ &= \$3790 \end{aligned}$$

(b) The annual equipment can be calculated from

$$\begin{aligned} A &= G [A/G, 15\%, 4] \\ &= G \left[ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right] \\ &= \$1000 \left[ \frac{1}{0.15} - \frac{4}{(1+0.15)^4 - 1} \right] \\ &= \$1000 * 1.3263 = \$1326.30 \end{aligned}$$

# The cash flow as follows:

End of year	Cash flow/\$
-------------	--------------

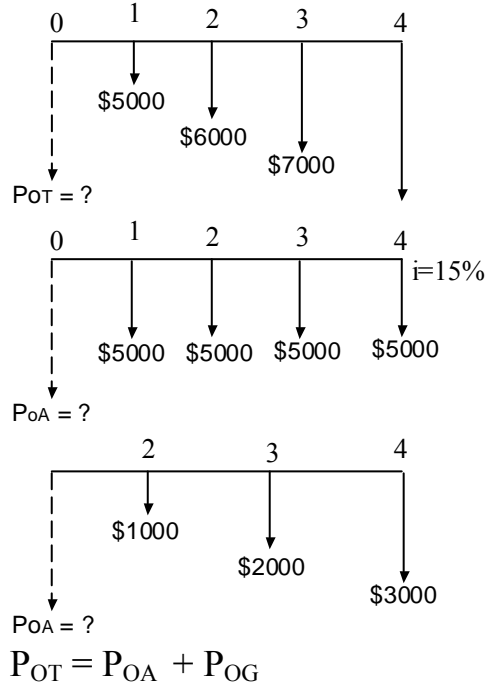
1	-5000
2	-6000
3	-7000
4	

# The cash flow as follows:

End of each year	cash flows(\$)
1	-8000
2	-7000
3	-6000
4	-5000

Calculate the present equivalent at  $i = 15\%$

Solution:



$$= A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + G \left[ \frac{(1+i)^n - 1 - ni}{i^2(1+i)^n} \right]$$

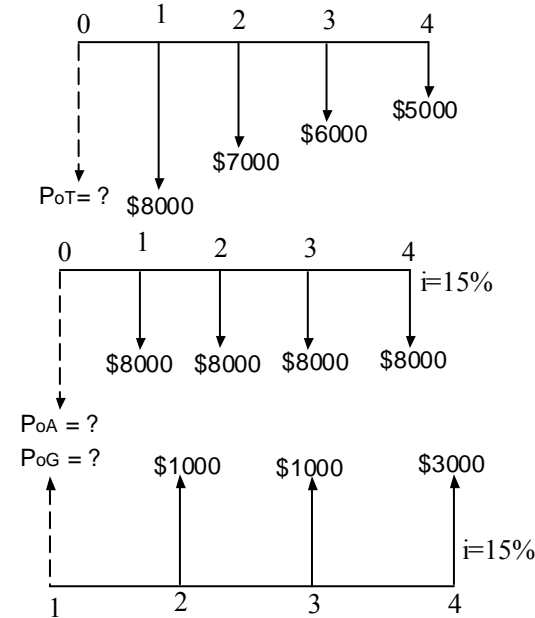
$$= -\$5000 \left[ \frac{(1+0.15)^4 - 1}{-0.15(1+0.15)^4} \right] + \$1000 \left[ \frac{(1+0.15)^4 - 1 - 4 \times 0.15}{0.15^2(1+0.15)^4} \right]$$

$$= -\$5000(2.8550) - \$1000(3.74)$$

$$= -\$14275 - \$3790 = -\$18065$$

Calculate the present equivalent at  $i = 15\%$

Solution:



$$P_{OT} = p_{OA} - p_{OG}$$

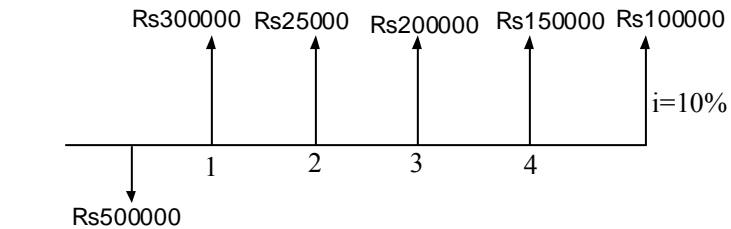
$$= -A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + G \left[ \frac{(1+i)^n - 1 - ni}{i^2(1+i)^n} \right]$$

$$= \$8000 \left[ \frac{(1+0.15)^4 - 1}{0.15(1+0.15)^4} \right] + \$1000 \left[ \frac{(1+0.15)^4 - 1 - 4 \times 0.15}{0.15^2(1+0.15)^4} \right]$$

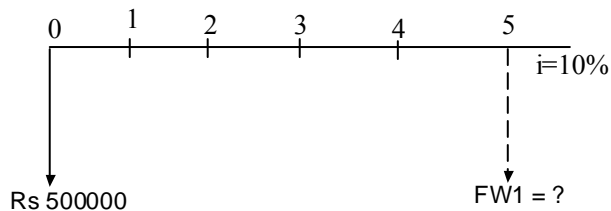
$$= -\$19050$$

# (5) The first investment cash for a projects is Rs 500000. The net annual revenue from the end of first years onwards are 3,00, 000, 2,50,000, 2,00,000, 1,50,000 and so on i.e decreasing by an amount of Rs 50,000 each year. Find out the FW of this cash flows at the end of 5 years.  $i = 5\%$ .

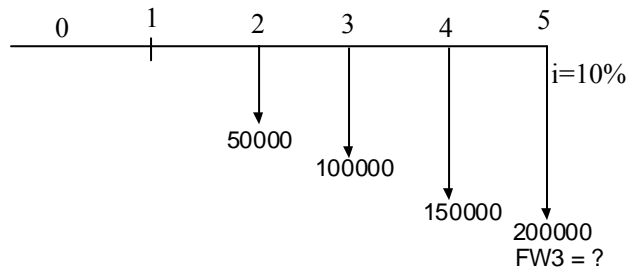
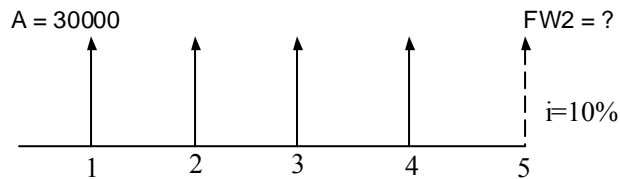
Solution:



Equal to (=)



+



Given ,  $i = 10\%$  ,  $n = 5$  yrs,  $G = Rs 50,000$  ,  $A = Rs 3,00,000$   
 $P = Rs 500,000$

$$\begin{aligned}
 Fw_T &= FW_1 + FW_g + FW_3 \\
 &= -p(1+i)^n + A \left[ \frac{(1+i)^n - 1}{i} \right] - \left[ \frac{G}{i} \left\{ \frac{(1+i)^n - 1}{i} \right\} - \frac{nG}{i} \right] \\
 &= \qquad \qquad \qquad \$5,00000(1+0.01)^5 \qquad \qquad \qquad + \\
 &Rs300000 \left[ \frac{(1+0.01)^5 - 1}{0.01} \right] - \left[ \frac{50000}{0.01} \left\{ \frac{(1+0.10)^5 - 1}{0.10} \right\} - \frac{5.50000}{0.10} \right]
 \end{aligned}$$

(6) What is the efficient interest rate if the nominal rate is 9% per year, a 365 day year is used and the compounding period is (a) yearly (b) quarterly (c) daily (d) hourly

Solution:

$$R = 9\% , i = ?$$

(a) When the compounding period is yearly

$$\begin{aligned}
 i &= (1+r/m)^m - 1 \\
 &= (1 + 0.09/1)^1 - 1 \\
 &= 0.09 \\
 &= 9 \%
 \end{aligned}$$

(b) When compounding period quarterly.

$$\begin{aligned}
 i &= (1 + r/m)^m - 1 \\
 &= (1 + 0.09/4)^4 - 1
 \end{aligned}$$

(c) When the compounding period is daily

$$\begin{aligned}
 i &= (1+r/m)^m - 1 \\
 &= (1 + 0.09/365)^{365} - 1
 \end{aligned}$$

(d) when compounding period is hourly

$$\begin{aligned}
 i &= (1+r/m)^m - 1 \\
 &= (1+0.09/8760)^{8760} - 1
 \end{aligned}$$

(7) Solve problem 2,3 and 4 assuming continuous compounding.

will earn 10% per year or more on all invested capital. Show whether this is desirable investment by using the PW method.

Date: 2065/6/3

Figure:

#### 4. BASIC MEHODOLOGIES OF ENGINEERING ECONOMICS STUDIES:

##### 4.1 Present worth and annual worth method:

**Present worth method:** It is based on the concept to equivalent worth of all cash flows relative to some base or beginning point in time called the preset, i.e all cash flows( inflow and outflows)are discounted to the base point of MARR .

PW is a measure as to how much money will have to be put aside how to provide for future expenditures.

$$PW = F_0 (1+i)^0 + F_1(1+i)^{-1} + F_2(1+i)^{-2} + \dots + F_k (1+i)^{-k} + F_n(1+i)^{-n}$$

- Where, i = effective interest rate or MARR
- K = index for each compounding period
- F<sub>k</sub> = Future cash flow.
- n = no of compounding periods

The formula is for constant interest rate. If i changes the calculation has to be done in steps. Higher the interest rate and further into the future a cash flow occurs, lower its present. As long as PW is greater then or equal to zero, the project is economically justified.

*# An investment of \$1000 can be made in a project that will produce a uniform annual revenue of \$5310 for five years and then have a market (salvage) value of \$2000. Annual expenses will be \$3000 each year. The company is will to accept any project that*

$$\begin{aligned}
 PW(10\%) &= \text{cash inflow} - \text{cash outflow.} \\
 &= \text{Annual revenue} + \text{Salvage value} - \text{investment} - \text{annual expenses.} \\
 &= \$5310(p/A, 10\%, 5) + \$2000(p/F, 10\%, 5) - \$10,000 - \$300(p/A, 10\%, 5) \\
 &= \$5310 \left[ \frac{(1 + 0.01)^5 - 1}{0.01(1 + 0.10)^5} \right] + \$2000(1+0.01)^{-5} - \$1000 - \\
 &\quad \$3000 \left[ \frac{(1 + 0.01)^5 - 1}{0.01(1 + 0.10)^5} \right] \\
 &= \$20125 + \$1245 - \$1000 - \$11370 \\
 &= 0, \text{ The project is shown to be marginally accepted.}
 \end{aligned}$$

**Annual worth method:** AW of a project is uniform annual series of amounts for a stated study period, which is equivalent to the cash inflows ( receipts or saving) and or cash outflow (expenses) under consideration. In other words AW of a project is its annual equivalent receipts (R) – annual equivalent expenses (E), less its annual equivalent capital recovery (CR) amount. R,E, CR is computed as MARR. Study period is denoted by an ‘n’ which is usually in years.

As long as AW is greater then or equal to zero the project is economically attractive. AW is zero means annual return equal to MARR has be earned.

CR for a project is the equivalent uniform annual cost of capital invested. It is the annual amount that covers

- (ii) Depreciation i.e loss in the value of assets

(iii) Interest on invested capital. (MARR)

CR can be easily calculated by finding annual equivalent of initial investment and then subtracting annual equivalent of salvage value

$$CR = I (A/p, i\%,n) - S (A/F, i\%,n)$$

Where

i = initial investment for a period.

S = Salvage value at the end of the study period.

n = project study period.

*# An investment of \$1000 can be made in a project that will produce a uniform annual revenue of \$5310 for five years and then have a market (salvage) value of \$2000. Annual expenses will be \$3000 each year. The company is will to accept any project that will earn 10% per year or more on all invested capital. Show whether this is desirable investment by using the PW method.*

Figure:

$$\begin{aligned} AW(\%) &= R-E-CR \\ &= \$5310 - \$3000 - [ \$1000(A/P, 10\%, 5) - \$2000(A/F,10\%, 5) ] \\ &= \quad \$ \quad 5310 \quad -\$ \quad 3000 \quad - \quad \left[ \$1000 \frac{(1+0.10)^5 \times 0.10}{(1+0.10)^5 - 1} \right] - \$ \\ &\quad 2000 \left[ \$1000 \frac{(1+0.10)^5 \times 0.10}{(1+0.10)^5 - 1} \right] \\ &= -\$5310 - \$3000 - [\$2638 - \$32800] \\ &= 0 \end{aligned}$$

The project is shown to be marginally acceptable.

**4.2 Future Worth method:** In this method all cash inflow and outflow are compounded forward to a reference point in the time

called future, to obtained the future worth. If  $FW \geq 0$  the project is feasible. If there are several alternative the one with maximum FW is chosen.

$$FW = F_0 (1+i)^n + F_1(1+i)^{n-1} + F_2(1+i)^{n-2} + \dots + F_k (1+i)^{n-k} + F_n(1+i)^0$$

Date: 2065/6/10

### 4.3 Internal rate of return (IRR) Method:

In this method that interest rate is found out that equates the equivalent worth of an alternative's Cash inflows ( receipts or savings) to the equivalent worth of cash outflows (expenditure or investments). The interest rate  $i'\%$  is calculated at which  $\sum_0^N R_k$

$$(p/F, i'\%, k) = \sum_0^N E_k (p/F, i', k) \text{ by using present worth}$$

formulation.

For a single alternative , IRR is not positive unless both receipts and expenses are present in the cash flow pattern and the sum of receipts exceeds the sum of all cash outflows cash flow pattern and the sum of receipts exceeds the sum of all cash outflows.

$R_k$  = net receipts for  $k^{\text{th}}$  year.

$E_k$  = net expenses including investment for the  $k^{\text{th}}$  year.

N = project life.

Once  $i'\%$  is calculated it is compared with MARR. If  $i'\% \geq$  MARR , the alternative is acceptable, otherwise not.

Alternative form

$$\sum_0^N R_k (p/F, i'\%, k) - \sum_0^N E_k (P/F, i'\%, k) = 0$$

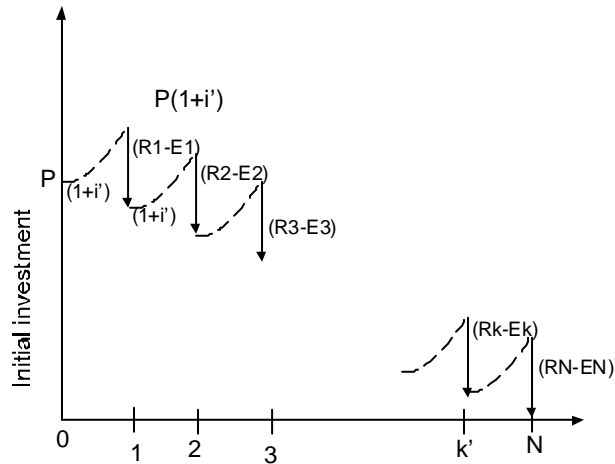
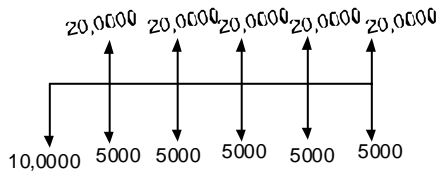
For an alternative with a single investment at the present time ( $k = 0$ ) followed by a series of +ve cash flows to  $N$ , the graph is shown. The point at which  $pw = 0$ ,  $i'$  is IRR.

The value of  $i'$  can also be determined as the interest rate at which  $FW = 0$  or  $AW = 0$ .

$$\sum_0^N R_k(F/P, i', N-K) - \sum_0^N E_k(F/P, i', N-k) = 0 \text{ using FW}$$

formulation.

**Another interpretation is that of unrecovered investment balance:**

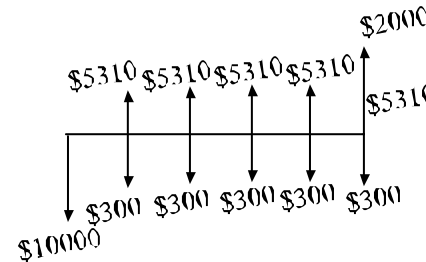


The figure shows how much of the original investment .....to be recovered as a function of time. Downward arrow indicates returns

,  $R_k - E_k$  for  $0 < k < N$  against unrecovered investment and dashed line represents opportunity cost of interest or profit on the beginning of year investment balance. IRR is that value of  $i'$  that can causes the unrecovered investment balance to exactly equal to zero at the end of study period ( $N$ ) and represent the interval earning of a profit.

Date: 2065/6/13

# Example for IRR method.



$$PW = 0 = -\$10,000 + (\$5310 - \$3000) (P/A, i', 5) + \$2000(P/F, i', 5)$$

$$\text{Or } Pw = 0 = -\$10000 + \$2310 \left\{ \frac{((1+i')^5 - 1)}{i'(1+i')^5} \right\} + \$200(1+i')^{-5}$$

Now by trail and error

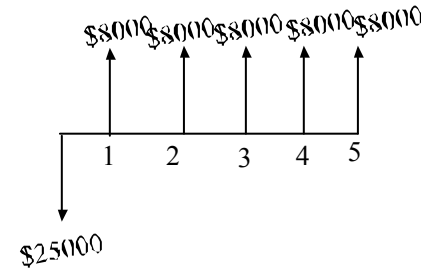
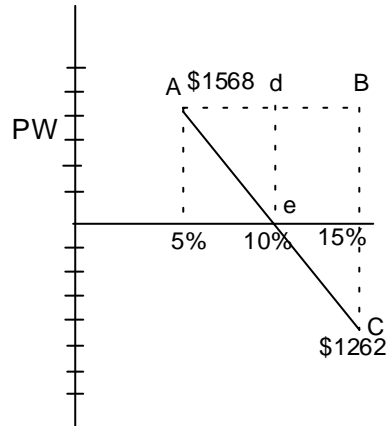
At  $i' = 5\%$ ,

$$Pw = -\$10000 + \$2310 \left\{ \frac{((1+0.05)^5 - 1)}{0.05(1+0.05)^5} \right\} + \$2000(1+0.05)^{-5}$$

$$= -\$10000 + \$2310(4.3295) + \$2000(0.7835) = +\$1568$$

At  $i' = 15\%$ ,

$$Pw = -\$10000 + \$2310(3.3522) + \$2000(0.4972) = -\$1262$$



$$P_w = 0 = -\$25000 + \$800(P/A, i', 5) + \$5000(p/F, i', 5)$$

$$\text{Or } P_w = 0 = -\$25000 + \$8000 \left\{ \frac{((1+i')^5 - 1)}{i'(1+i')^5} \right\} + \$5000(1+i')^{-5}$$

**By trial and error:**

$i'$ %	PW
10%	$-\$25000 + \$8000(3.7908) + \$5000(0.6209) = \$830.90$
20%	$-\$25000 + \$8000(2.9908) + \$5000(0.4019) = \$934.30$
25%	$-\$25000 + \$8000(2.6893) + (\$5000 \times 0.3277) = -\$1847.10$

**By interpolation :**

$$i' \% = 20\% + [\$934.30 / (\$934.30 + \$1897.10)] \cdot (25\% - 20\%)$$

$$= 20\% + 1.7\%$$

$$= 21.7\%$$

For exact value of  $i'$ %

$$i' = 21.577\% \text{ i.e } p.w = 0$$

$$\therefore \text{IRR} = 21.577\% > \text{MARR}$$

$\therefore$  This investment is a good one .

#### 4.4 Drawbacks of IRR method:

- The IRR method is based on assumption that recovered fund, if not consumed in each time period are reinvest at  $i'$ % rather than at MARR. This is not always practical.

Line AB/line BC = Line dA/line de

$$15\% \cdot 5\% / \$1568 - (-1262) = i' \% \cdot 5\% / \$1568 - 0$$

$$i' \% = 5\% + \$1568 / (\$1568 + \$1262) \cdot (15\% - 5\%)$$

$$= 5\% + 5.5\% = 10.5\%$$

$$\therefore P_w(10\%) = -\$10000 + \$2310(3.7908) + 2000(0.6209)$$

$$= 0$$

$$\therefore \text{IRR} = 10\%$$

# A piece of new equipment has been purposed by Engineers to increase the productivity of a certain manual welding operation. The investment cost is \$25000 and equipment will have a salvage value of \$5000 at the end of its expected life of five years. Increased productivity attribution to the equipment will amount to \$8000 per year after extra operating cost have been subtracted from the value of additional production. Evaluate the IRR of purpose equipment. Is the investment a good one? Recall the MARR is 20% per year.

Cash flow diagram :



b. Sometimes it may not be uniquely defined. If the cash flow stream of a project has more than one change in sign, there is a possibility of multiple rates of return.

Year	cash
0	-1600
1	+1000
2	-1000

$$P_w = 0 = -1600 + 1000(1+i')^{-1} - 1000(1+i')^{-2}$$

$$i' = 25\% \text{ and } 400\%$$

Both of them are incorrect.

c. IRR method can be misleading when choosing between mutually exclusive project that have sequentially different outlays. Consider project P and Q

Project	Cash flow		IRR	NPV at $i = 12\%$
	0	1		
P	-10000	+10000	100%	7857
Q	-50000	+75000	50%	16964

Both projects are good but Q with it higher NPV worth more to the stockholders. But for IRR point of view P looks better. Hence IRR criteria is unsuitable for ranking project of different scale.

Date: 2065/7/20

#### 4.4 External Rate of Return method (ERR):

The reinvestment assumption of IRR may not be valid sometimes for example, if a firm's MARR is 20% per year and IRR of a project is 40% , it may not be possible for the firm to reinvest net cash proceeds from the project at much more than 40%. ERR method eliminates this drawback to some extent. ERR method takes into account the external interest rate (E) at which net cash flow generated by a project over its life can be reinvested outside

the firm. In general all cash outflow are discounted to period 0(present) at E% per compounding period while all cash inflow are compounded to period n at E%. The ERR is then the interest rate that establishes equivalence between the two quantities , i.e ERR is the  $i'$ % at which

$$i' \% = ? \quad \sum_{k=0}^n R_k (F/p, E\%, n-k)$$

$$\sum_{k=0}^n E_k (P/F, E\%, k)$$

$$\sum_{k=0}^n E_k (P/F, E\%, k) = \sum_{k=0}^n R_k (F/p, E\%, n-k)$$

A project is acceptable when  $i'$ % of ERR method is greater then or equal to firm's MARR.

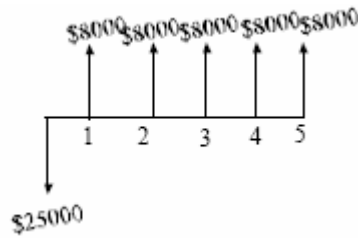
#### Advantage:

1. It does not need trial and error to solve  $i'$ % .
2. There is no possibility of multiple rate of return.

Example:

# A piece of new equipment has been purposed by Engineers to increase the productivity of a certain manual welding operation. The investment cost is \$25000 and equipment will have a salvage value of \$5000 at the end of its expected life of five years. Increased productivity attribution to the equipment will amount to \$8000 per year after extra operating cost have been subtracted from the value of additional production. Evaluate the IRR of purpose equipment. Is the investment a good one? Recall the MARR is 20% per year.

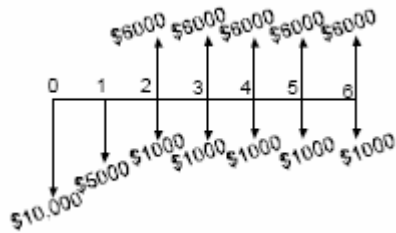
Cash flow diagram :



$$\begin{aligned} \$25000 (F/p, i', 5) &= \$8000(F/A, 20\%, 5) + \$5000 \\ \$25000 \{(1+i)'\}_5 &= \$8000 \{[(1+0.20)^5 - 1]/0.20\} + \$5000 \\ \text{Or, } \$25000(1+i')^5 &= \$64532.80 \\ \text{Or, } (1+i) &= 2.5813 \\ i' &= 20.88\% \gg \text{MARR (project is ok)} \end{aligned}$$

Date: 2065/7/22

# When E=15% and MARR is 20% per year determine whether the project whose cash flow diagram appears below is acceptable.



Here;

$$\begin{aligned} \{ \$1000 + \$5000(1+0.15)^{-1} \} (1+i')^6 &= (\$6000 - \$1000)(F/A, E\%, 5) \\ \text{Or, } 14347.83(1+i')^6 &= \$5000 \{ [(1+0.15)^5 - 1]/0.15 \} \\ \text{Or } 14347.83(1+i')^6 &= 33711.90 \\ \therefore i' &= 14.2 < 20 \text{ (so project is not acceptable)} \end{aligned}$$

#### 4.5 Minimum Attractive Rate of Return method: (MARR)

The minimum attractive rate of return is usually is a policy issue resolved by top management of an organization in view of numerous consideration. This consideration are as follows:

1. The amount of money available for the investment and the source and cost of this fund (i.e equity fund or borrowed fund)
2. The number of good projects available for investment and their purpose (i.e whether they sustain present operations and are essential or expand on present operations)
3. The amount of perceived risk associated with investment opportunities available to the firm and the estimated cost of the projects (short planning verses long planning).
4. The type of the organization involved (i.e government, public utility or competitive industry)

In theory the MARR, which is some times called the Hurdle rate, should be chosen to maximize the economic wellbeing of an organization one popular approach to establish a MARR involves the opportunity cost view point.

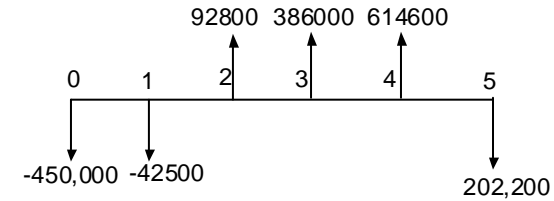
**4.7 Payback (Payout) period method:** The payback period method mainly indicates a project's liquidity rather than its profitability. The payback method deals with how fast an investment can be recovered. The payback methods calculate number of year required for cash inflows to equal cash outflows. Simple payback period ignores the time vale of money. Some times discounted payback period is calculated which considers time value of money.

$$\begin{aligned} \sum_{k=0}^{\theta} (R_k - E_k) - I &\geq 0 \text{ i.e Simple payback period.} \\ \sum_{k=0}^{\theta} (R_k - E_k)(p/F, i\%, k) - I &\geq 0 \text{ i.e Discounted payback period.} \end{aligned}$$

#### Drawbacks:

1. The most serious deficiencies of payback period are that it fails to consider the time value of money.

2. The Consequences of the investment following the payback period, including the magnitude and timing of the cash flows and the expected life of the investment.



Date: 2065/7/25

#Calculation of Simple payback period and discounted payback period of MARR = 20%.

End of year	Net cash flow	Pw at i = 0%	Pw at i = 20%
0	-25000	-25000	-25000
1	8000	-17000	-18333
2	8000	-9000	-12777
3	8000	-1000	-8147
4	8000	7000	-4289
5	13000	20000	934

For I = 0% , payback period = 4 years (At the end of 4<sup>th</sup> year the balance turns +ve).

For I = 20% payback period = 5 years (At the end of 5<sup>th</sup> year the balance turns +ve)

**Q.1.** Find IRR for the following project.

End of year	Net cash flow
0	-450,000
1	-42500
2	+92,800
3	386,000
4	614,600
5	-202,200

Solution:

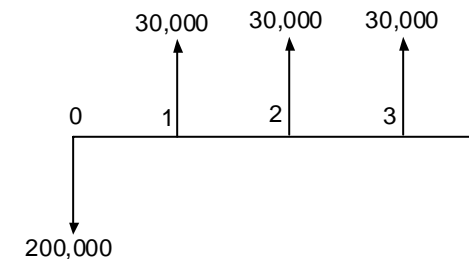
By using IRR Method,

$$PW = -450000 - 42500(p/F, i', 1) + 92800(P/F, i', 2) + 386000(P/F, i', 3) + 614600(P/F, i', 4) - 202200(P/F, i', 5) = 0$$

$$\text{Or, } -450000 - 42500(1+i')^{-1} + 92800(1+i')^{-2} + 386000(1+i')^{-3} + 614600(1+i')^{-4} - 202200(1+i')^{-5} = 0$$

**Q.2.** Find IRR for the given cash flow using AW method. Also show the unrecovered investment balance in the tabular form as well as in the graphical form.

Year	Cashflow
0	-20,000
1	+30000
2	+30000
3	+30000



$$AW = R - E - CR = 0$$

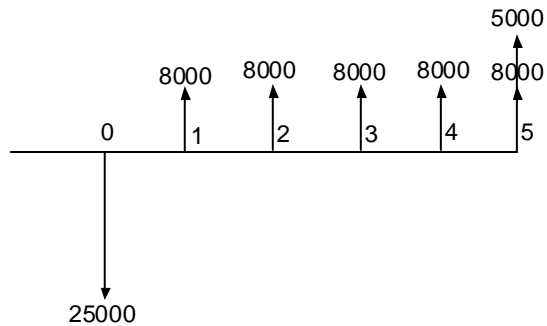
$$AW = 30,000 - 200,000(A/P, i', 3) = 0$$

$$\text{Or, } 30,000 - 200,000 \left[ \frac{\{(1+i')^3 i'\}}{(1+i')^3 - 1} \right] = 0$$

**Q.3.** What do you mean by external rate of return method?

Calculate ERR for the following project of  $E = 15\%$  per year.

Year	Cashflow
0	-25000
1	8000
2	8000
3	8000
4	8000
5	13000

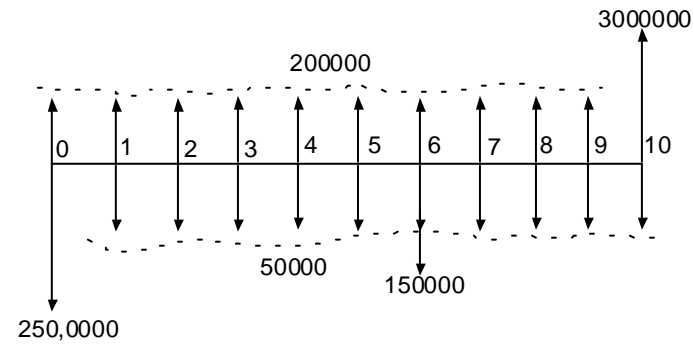


$$25000(F/p, i', 5) = 8000(F/A, E\%, 5) + 5000$$

$$\text{Or, } 25000(1+i')^5 = 8000 \left[ \frac{(1+0.15)^5 - 1}{0.15} \right] + 5000$$

$$\text{Or, } i' = 18.7\%$$

**Q.4.** A man purchased a building 10 year ago for Rs. 25,00,000. Its maintenance cost is Rs.50,000 per year. At the end of six years he spent Rs.150,000 on roof repairs. After the end of 10 years he sold the building for Rs.3000000. During the period of ownership, he put the building on rent for Rs. 20,0000 per year paid at the beginning of each year. Use PW AW and FW methods to evaluate this investment when his MARR is 12% per year.



**Date: 2065/7/27**

### 5. Cost/Benefit analysis:

The objective is to learn how to use the benefit/cost (B/C) ratio method as a criterion for the selection for the project .

As the name implies, the benefit/cost method involves the calculation of a ratio of benefits to costs. Whether evaluating a project in the private sector or in the public sector the time values of money must be considered to account for the timing of cash flows (or benefits) occurring after the inception of the project. Thus B/C ratio is actually a ratio of discounted benefit to discounted costs.

Any method for formally evaluating project in the public sector must be consider the worthiness of allocating resources to achieve social goals. For over 60 yrs , the B/C ratio method has been the accepted procedure for making go/no-go decisions on independent projects and for comparing alternative projects in the public sectors , even though the other methods. i.e PW, AW, IRR etc, will lead to identical recommendation, assuming all these procedures are properly applied.

The B/C ratio is defiend as the ratio of the equivalent worth of benefits to the equivalent worth of costs. The equivalent worth

measure applied can be present worth, annual worth or future worth but customarily Pw or AW is used.

Salvage value 50,000, annual benefit 85000, annual O&M cost 25000 & I = 15%.

**5.1 Conventional benefit/cost ratio:**

Conventional B/C ratio with PW formulation:

$$\begin{aligned} B/C &= PW(\text{Benefit of proposed project})/PW(\text{cost of proposed project}) \\ &= PW(B)/PW(C) \\ &= PW(B)/\{I+PW(O \& M)\} \dots\dots\dots(i) \end{aligned}$$

Where, I = Initial investment in project.

O& M = Operation & maintenance cost.

B = Benefits of project.

Conventional B/C ratio with AW,

$$\begin{aligned} B/C &= AW(\text{Benefit of the proposed project})/AW(\text{ Total cost of proposed project}) \\ &= AW(B)/\{CR+AW(O \& M)\} \end{aligned}$$

A project is accepted when B/C ratio as defined in the above equation is greater than or equal to 1.

**Date: 2065/7/29**

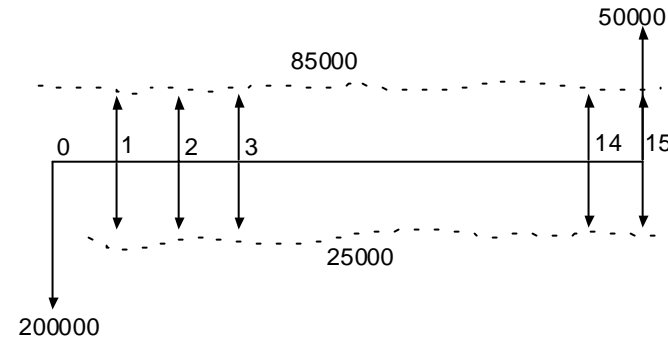
**5.2 Modified benefit/Cost ratio:** The numerator of the modified B/C ration expresses the equivalent worth of the benefits minus the equivalent worth of the O&M costs and denominator includes only the initial investment costs.

$$\therefore \text{Modified B/C ratio with PW} = \{PW(B) - PW(O\&M)\}/I \dots(i)$$

$$\begin{aligned} \text{Modified B/C ratio with P/W, salvage value included,} \\ &= \{PW(B) - PW(O\&M)\}/\{I-PW(s)\} \end{aligned}$$

$$\text{Modified B/C ratio with AW} = \{AW(B) - AW(O\&M)\}/CR$$

#. Find both types of B/C ratios using PW formulation for a project having first investment cost Rs. 200000, project life 15 yrs ,



$$\begin{aligned} \text{Conventional B/C ratio formulation} &= PW(B)/\{I- \\ &PW(s)+PW(O\&M)\} \\ &= 85000(p/A,5\%,15)/[20,000-50000 \\ &(p/F,15,15)+25000(P/A,15\%,15)] \end{aligned}$$

Modified B/C ratio with PW formulation:

$$B/C = PW(B)PW(O\&M)/\{I - PW(s)\}$$

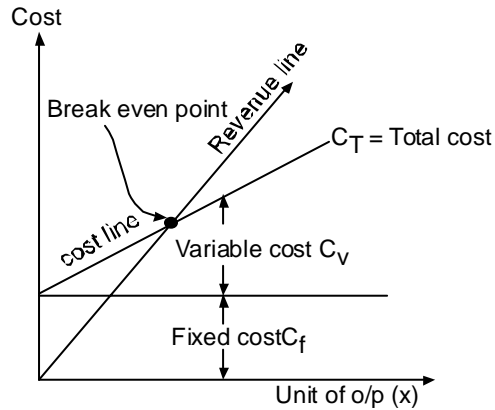
**5.3 Breakdown analysis:**

Basic Concept:

$$C_T = C_F + C_v(x)$$

$C_v(x)$  = variable cost per unit of o/p

$$S_T = S_p(x) , S_p = \text{selling price per o/p}$$



The breakeven principal can be used for the following parameters:

- (i) **Revenue & annual cost:** Solve for annual revenue required to breakdown with annual costs. Then compare between alternatives.
- (ii) **Rate of return:** Solve for the rate of return at which two given alternatives are equally desirable.
- (iii) **Salvage value:** Solve for future resale that would result in indifference as to preference for an alternative.
- (iv) **Equipment life:** Solve for the hour of utilization per year at which an alternatives are equally desirable.

Through breakeven analysis one can solve for the value of parameter at which the conclusion is stand off. That value is breakeven point.

**Example:** 100 HP

	Motor A	Motor B
Purchase cost	125000	1,60,000
$\eta$	74%	92%
Life	10 yrs	10 yrs
Maintenance cost	5000/yrs	2500 yrs

Annual tax and insurance = 1 ½ % of investment

MARR = 15%

How many hours/yr would the motors have to be operated at full load for load for annual costs to be equal.

Electricity cost = Rs.5/hrs.

**Motor A:**

$$\begin{aligned} \text{CR cost} &= 125000 (A/P, 15, 10) \\ &= 125000 [ (0.15 * 1.15^{10}) / (1.15^{10} - 1) ] \\ &= 24906.50 \end{aligned}$$

Operating cost for power: =  $(100 * 0.746 * 5x) / 0.75 = 504x$

Maintenance cost = 5000/yrs

Tax and insurance cost =  $0.015 * 125000 = 1875$

Total Annual cost =  $24906.50 + 504x + 1875 + 5000$

**Motor B:**

$$\begin{aligned} \text{CR cost} &= 160000 (A/p, 15, 10) \\ &= 160000 [ (0.15 * 1.15^{10}) / (1.15^{10} - 1) ] \\ &= 31880.32 \end{aligned}$$

O.P cost =  $(100 * 0.746 * 5x) / 0.92 = 405.43x$

Tax and insurance cost =  $160000 * 0.015 = 2400$

Maintenance cost = 2500

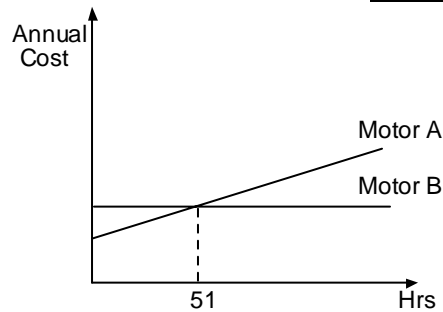
$\therefore$  Total annual cost =  $31880.32 + 405.43x + 2400 + 2500$

Now, According to given condition,

Annual cost of Motor A = Annual cost of Motor B.

$\therefore 24906.50 + 504x + 1875 + 5000 = 31880.32 + 405.43x + 2400 + 2500$

$\therefore x = 50.713 \approx 51$  hrs.



Date: 2065/8/4

<6.0> Investment Decision:

<6.1> Comparison of alternative having useful life:

- (a) Equivalent worth method: (pw, FW, AW)
- (b) Rate of return methods (IRR, ERR)

(a) Equivalent worth methods:

# Study period = 10 yrs.

MARR = 10%

Useful life = 10 yrs

Salvage value = 0

Which alternative should be selected?

	Alternative		
	A	B	C
Investment cost	390000	920000	660000
Net annual receipts less expenses	69000	167000	133500

PW method:

$$PW_A(10\%) = -390000 + 69000(p/A, 10\%, 10) = 33975$$

$$PW_B(10\%) = -920000 + 167000(P/A, 10\%, 10) = 106143$$

$$PW_C(10\%) = -660000 + 133500 + (p/A, 10\%, 10) = 160300$$

Therefore, order of preference is C>B>A

AW method:

$$AW_A(10\%) = R-E-CR = 69000 - 390000(A/P, 10\%, 10) = 5529.00$$

FW method:

$$FW_A(10\%) = -390000(F/p, 10\%, 10) + 69000(F/A, 10\%, 10) = 88122$$

# When estimated costs only are relevant:

	A	B	C	D
Investment	6000	7600	12400	13000
Useful life	5	5		5
Annual O & M				
Power	680	689	1200	1260
Labour	6600	6000	4200	3700
Maintenance	400	450	650	500
Tax and insurance	120	152	248	260
Total annual cost	7800	7282	6298	5720
i = 10%				
Pw of investment	6000	7600	12400	13000
Total annual cost	29568	27605	23874	21683
(p/A, 10%, 5)				
	-35568	-35205	-36274	-34683

The order of preference:

D>B>A>B

Aw method:

O& M	-7800	7282	6298	5720
CR	-1583	-2005	-3271	-3429
	-9383	-9287	-9569	-9143

Order of preference is same.

Date: 2065/8/9

End of period	A	B	C
0	-640000	-680000	-755000
1	2620000	-40000	205000
2	290000	392000	406000
3	302000	380000	400000
4	310000	380000	390000
5	310000	380000	390000
6	260000	380000	324000

**Rate of Return Method:**

	A	B	C	D	E	F
Inv. Cost	900	1500	2500	4000	5000	7000
Annual.Rev.	150	276	400	925	1125	1425
Less cost.						

N = 10 yrs

S = 0

MARR = 10 %

Solution:

AW formulation ,  $0 = -900(A/P, i', 10) + 150$   
 $i' = 10.6\%$

IRR on	A	B	C	D	E	F
Tot. inv.	10.6	13	9.6	19.1	18.3	15.6

C is rejected as its IRR is below 10%

Increment Consider	A	A-B	B-D	D-E	E-F
Δ inv.cost	900	600	2500	1000	2000
Δ Annual rev. less cost	150	126	649	200	300
IRR on Δ inv.cost	10.6%	16.4%	22.6%	15.1%	8.1%
Is increment justified	yes	yes	yes	yes	No.

Hence the project is better.

# Using ERR method.

N = 6yrs

MARR = 20%

E = MARR

Figure:

Solution:

For project A:

$$640000(F/P, i', 6) = 262000(F/P, 20\%, 5) + 290000(F/P, 20\%, 4) + 302000(F/P, 20\%, 3) + 310000(F/P, 20\%, 2) + 310000(F/P, 20\%, 1) + 260000(F/P, 20\%, 0)$$

∴  $i'_A = \dots\dots$

For project B:

$$\{680000 + 40000(P/F, 20\%, 1)\}(F/P, i', 6) = 392000(F/P, 20\%, 4) + 380000(F/P, 20\%, 3) + 380000(F/P, 20\%, 2) + 380000(F/P, 20\%, 1) + 380000$$

$i'_B = \dots\dots$

For Project C:

$$755000(P/F, i'_c, 6) = 205000(F/P, 20\%, 5) + 406000(F/P, 20\%, 4) + 400000(F/P, 20\%, 3) + 390000(F/P, 20\%, 2) + 390000(F/P, 20\%, 1) + 324000(F/P, 20\%, 0)$$

∴  $i'_c = \dots\dots$



Incremental cash flow:

A	A-B	A-C
-640000	-40000	-115000
262000	-302000	-57000
290000	102000	116000
302000	78000	98000
310000	70000	80000
310000	7000	80000
260000	120000	64000

	A	A-B	A-C
ERR on $\Delta$ cash flow	28.3%	14.3%	27%
Is increment is justified	yes	no	yes

$\therefore$  C is preferred.

### 6.2 Comparison of alternatives having different useful life:

When the useful lives of mutually exclusive alternatives are different,

- (a) Repeatability assumption may be used in their comparison of the study period can be infinite in length or a common multiple of the useful lives.
- The economic consequence that are estimated to happen in an alternative's initial life span will also in all succeeding life span (replacement).
  - Actual situation in engineering practice seldom meet this condition. This has tended to limit the use of the repeatability assumption, except in those situation where difference between the annual worth of the first life cycle and the annual worth over more than one cycle of the assets involved is quite small.

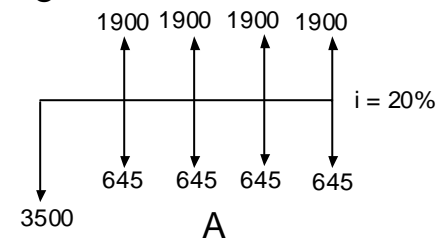
(b) Coterminated assumption may be used in their comparison if study period is finite and identical. This is the approach most frequently used in engineering practice because product life cycle are becoming shorter.

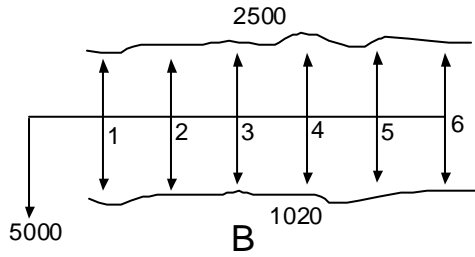
- To force a match of cash flow durations to the cotermination time, adjustment are made to cash flow estimates of the project alternatives having useful life different from the study period.

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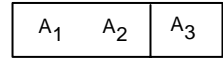
# The following data have been estimated for tow manually exclusives alternatives A and B , associated with a small engineering project for which revenues as well as expenses are involved. The y have useful live of 4 and 6 years respectively Of MARR = 10% per year, show which alternatives is more desirable by suing equivalent worth methods. Use the repeatability assumption

	A	B
Capital investment	3500	5000
Annual Revenue	1900	2500
Annual expenses	645	1020
Useful life(yrs)	4	6
Salvage value	0	0

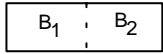




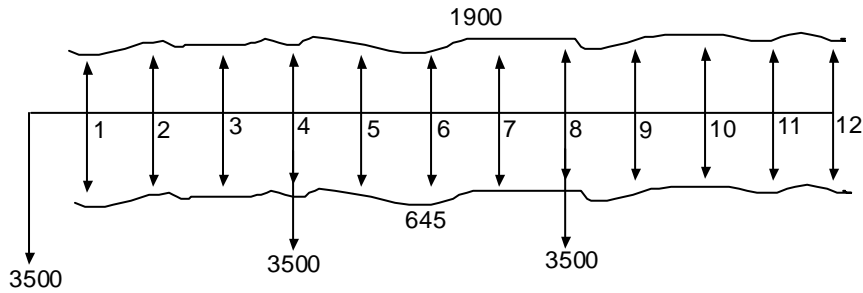
The least common multiple of the useful lines of alt A and B is 12.



Three cycle of Alt A.



Two cycle of Alt B.



Using PW method:

$$PW(10\%)_A = -3500 - 3500(P/F, 10\%, 4) - 3500(P/F, 10\%, 8) + (1900 - 645)(P/A, 10\%, 12) = 1028$$

$$PW(10\%)_B = -500 - 500(P/F, 10\%, 6) + (2500 - 1020)(P/A, 10\%, 12)$$

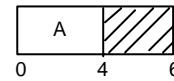
Since,  $PW(10\%)_B > PW(10\%)_A$

Therefore choose alt B.

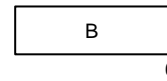
Date: 2065/8/13

# Suppose in the above example the expected period of required service for A and B is only 6 yrs. MARR = 10%, Which alternative is more desirable using co-terminated assumption.

Solution:



Assumed reinvestment of the cash flow at the MARR for tow years.



When useful life is less than the study period all cash flow will be reinvested by the firm of MARR until the end of study period. This will apply to alt. A.

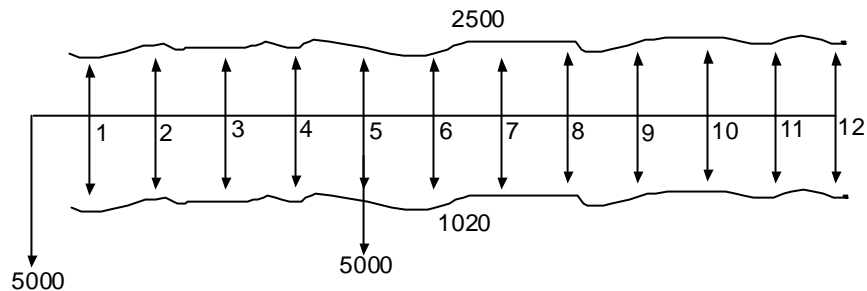
$$FW(10\%) = [ -3500(F/P, 10\%, 4) + (1900 - 645) F/n, 10\% 4 ] (F/p, 10\%, 2) = 847$$

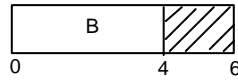
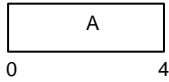
$$PW(10\%)_B = -500 (F/P, 10\%, 6) + (2500 - 1020) (F/A, 10\%, 6) = 2561$$

$FW(10\%)_A < FW(10\%)_B$

Choose alternative B.

# Suppose in the above example the expected period of required service for A and B is only 4 years. MARR= 10% which alternative is more desirable using coterminated assumption?





Assume salvage value of the cash flow for 2 years of MARR at the end of 4 years .

Salvage value of 4 years,

$$S_4 = [ 5000(A/P, 10\%, 6)] (P/A, 10\%, 2)$$

$$= X$$

$$AW(10\%)_A = R-E-CR = 1900-645-3500(A/P, 10\%, 4) =$$

$$AW(10\%)_B = R-E-CR = 2500-1020-\{5000(A/P, 10\%, 4) - x(A/F, 10\%, 4)\}$$

**<6.4> Comparison of alternative using capitalized worth method (CW method):**

Capitalized worth (CW) is the present worth of all receipts or expenses over an infinite length of time. This method of comparison is called CW method. If only expenses are Considered we called it capitalized cost method. This method is used for comparing mutually exclusive alternatives when period of service needed is indefinitely large or common multiple of lives is very long , and repeatability assumption is applicable.

CW of a perpetual series of end of period uniform payment A with interest at i% per period is  $A(P/A, i\%, \infty)$

Capitalized worth of A,

$$P = A(P/A, i\%, \infty)$$

$$= A \left[ \lim_{n \rightarrow \infty} \frac{(1+i)^n - 1}{i(1+i)^n} \right] = \frac{A}{i}$$

Annual worth of a series of payment of amount X at the end of k<sup>th</sup> period at int. rate i% is  $X(A/F, i\%, k)$

$$\therefore CW = [X(A/F, i\%, K)]$$

# MARR=15%

	M	N
I	12000	40000
S	0	40000
Annual expenses	2200	1000
Useful life	10	25
Assume infinite project life		

Solution:

	M	N
First cost	12000	40000
Replacements:		
12000(A/F, 15%, 10)/0.15	3940	
(40000-10,000)(A/F, 15%, 25)/0.15		940
Annual expenses	14667	
2200/0.15		
1000/0.15		6667
<b>Total capitalized worth (cost)</b>	<b>30607</b>	<b>47607</b>

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**6.3 Definition of Mutually exclusive investment alternatives in terms of combination of project:**

Category of investment opportunities:

(a) Mutually exclusive: at most one project out of group can be chosen.

- (b) Independent: the choice of a project independent of the choice of any other project in the group so that all or none of the project may be selected, or some number in between .
- (c) Contingent: Choice of a project is conditional on the choice of one or more other projects.

There may be different types of projects. So we must list feasible combination of projects and analyze them. Such combination will then be mutually exclusive. The net cash flow of each combination is determined simply by adding period by period the cash flow of each project included in the combination.

Suppose we have three projects: A,B,C. If all are mutually exclusive then following combination are obtained.

Mutually exclusive Condition	Project			
	XA	XB	XC	
1	0	0	0	Accept none
2	1	0	0	Accept A
3	0	1	0	Accept B
4	0	1	1	Accept C

Mutually exclusive combination of three independent project:

Mutually exclusive Condition	Project			
	XA	XB	XC	
1	0	0	0	Accept none
2	1	0	0	Accept A
3	0	1	0	Accept B
4	0	1	1	Accept C
5	1	1	0	Accept A or B
6	1	0	1	Accept A or C
7	0	1	1	Accept B or C
8	1	1	1	Accept A or C or B

Suppose A is contingent on acceptance of B and C, C is contingent on acceptance of B. The mutually exclusive combinations are,

- (a) Do nothing
- (b) B only
- (c) B and C
- (d) A , B and C

Next, two independent sets of mutually exclusive projects.  $A_1$  and  $A_2$  are mutually exclusive and  $B_1$  and  $B_2$  also same.

Mutually exclusive Combination	Project			
	$x_{A_1}$	$x_{A_2}$	$x_{B_1}$	$x_{B_2}$
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1
6	1	0	1	0
7	1	0	0	1
8	0	1	1	0
9	0	1	0	1

# Some of the projects are mutually exclusive.  $B_1$  and  $B_2$  are independent of  $c_1$  and  $c_2$  . Use PW method , MARR = 10%. Determine what combination of projects is best if capital to be invested is

- (a) Unlimited
- (b) Limited to 48000

$B_1$  and  $B_2$  mutually exclusive.

$C_1$  and  $C_2$  mutually exclusive contingent on acceptance of  $B_2$ .

D contingent on acceptance of  $c_1$

4	-44	16	16	16	16	-44	6.7
5	-45	17	17	17	171	-45	8.9
6	-54	22	22	22	22	-54	15.7
7	-64	24	24	24	24	-64	.....

C.F

Project	0	1	2	3	4
B <sub>1</sub>	-50	20	20	20	20
B <sub>2</sub>	-30	12	12	12	12
C <sub>1</sub>	-14	4	4	4	4
C <sub>2</sub>	-15	5	5	5	5
D	-10	6	6	6	6

### 6.6 Comparison of Independent project:

*Date: 2065/8/18*

### Chapter: 7

Mutually exclusive combination:

Mutually exclusive  
Combination

Project

	B <sub>1</sub>	B <sub>2</sub>	C <sub>1</sub>	C <sub>2</sub>	D
1	0	0	0	0	0
2	1	0	0	0	0
3	0	1	1	0	0
4	0	1	1	0	0
5	0	1	0	1	0
6	0	0	1	0	1
7	1	0	1	0	0

### 7.0 Risk analysis:

- Concept of Certainty.
- Concept of risk
- Concept of Uncertainty.

### 7.1 Project operation under certainty.

### 7.2 Project operating under conditions of uncertainty.

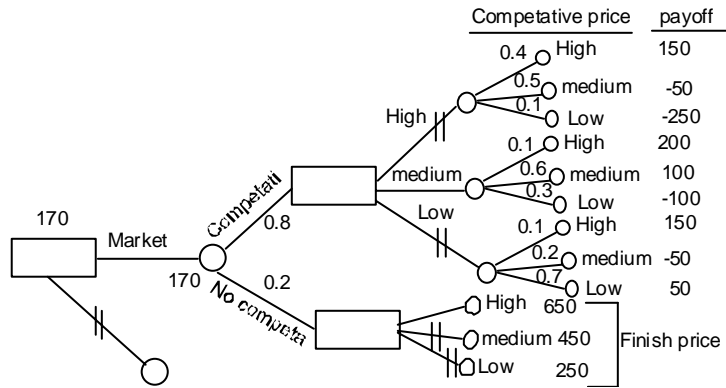
1. Break even analysis.
2. Sensitivity Analysis.
3. Optimistic-Pessimistic estimation.
4. Risk adjusted MARR
5. Reduce useful life.

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Combined project cash flow:

Mutually Exclusive	C.F(1000)					init.cap.	pw(10%)
	0	1	2	3	4		
1	0	0	0	0	0	0	0
2	-50	20	20	20	20	-50	13.4
3	-30	12	12	12	12	-30	8.0

### 7.3 Decision tree (Risk tree analysis):



$$(150 \cdot 0.4) + (-50 \cdot 0.5) + (-250 \cdot 0.1) = 60 - 25 - 25 = 10$$

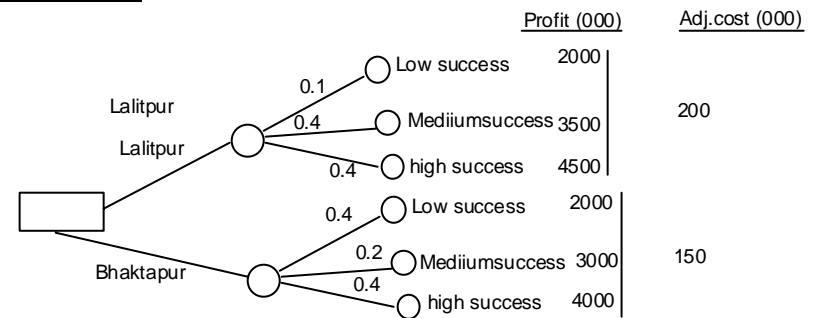
$$(200 \cdot 0.1) + (100 \cdot 0.6) + (-100 \cdot 0.3) = 20 + 60 - 30 = 50$$

$$(150 \cdot 0.1) + (50 \cdot 0.1) + (0.7 \cdot 0.5) = 15 + 10 - 35 = -10$$

Q. A ktm business form is considering the possibility of expanding its business to one of the two possible market area lalitpur or Bhaktapur. A preliminary analysis following data

	lalitpur		Bhaktapur	
	probability	Profit(000)	Probability	Profit(000)
Low success	0.20	2000	0.40	2000
Medi. Success	0.04	3500	0.20	3000
High success		4500	0.40	4000

The cost of advertising for lalitpur is Rs. 200000 and for Bhaktapur is Rs. 150000. Find out which market should be targeted by the firm.



$$(2000 \cdot 0.2) + (0.4 \cdot 3500) + (450 \cdot 0.4) - 200 = 3400$$

$$(2000 \cdot 0.4) + (3000 \cdot 0.2) + (4000 \cdot 0.4) - 150 = 2850$$

### 7.4 Sensitivity Analysis:

# Investigate the pw of the following project of a machine over a range of +40% in (a) initial investment (b) Annual net revenue (c) Salvage value and (d) Useful life.

Initial investment,  $I = 11,500$   
 Net annual revenue,  $A = 3000$   
 Salvage value,  $S = 1000$   
 Useful life,  $N = 6$  yrs  
 MARR = 10%

Draw also the sensitivity diagram.

$$Pw(10\%) = -11,500 + A(p/A, 10\%, 6) + S(P/F, 10\%, 6) = 2130$$

(a) When the initial investment varies by  $\pm P\%$ , the present worth is,

$$PW(10\%) = -11500(1 \pm P\%) + 3000(P/A, 10\%, 6) + 1000(p/F, 10\%, 6)$$

When I varies +40%

$$PW(10\%) = \dots\dots\dots$$

When I varies -40%

$$PW(10\%) = \dots\dots\dots$$

(b) When annual revenue changes by  $+a\%$ ,

$$PW(10\%) = -11,500 + 3000(1+a\%)(p/A, 10\%, 6) + 1000(p/F, 10\%, 6)$$

When  $a\%$  varies  $+40\%$

$$PW(10\%) = \dots\dots$$

When  $a\%$  varies  $-40\%$

$$PW(10\%) = \dots\dots\dots$$

(c) When salvage value varies by  $+s\%$ , the present worth,

$$PW(10\%) = -11,500 + A(p/A, 10\%, 6) + 1000(1+s\%)(p/F, 10\%, 6)$$

When  $s\%$  is  $+40$  varies

$$PW(10\%) \dots\dots\dots$$

When  $s\%$  is  $-40$  varies

$$PW(10\%) \dots\dots\dots$$

(d) when useful life changes by  $+n\%$ , Pw

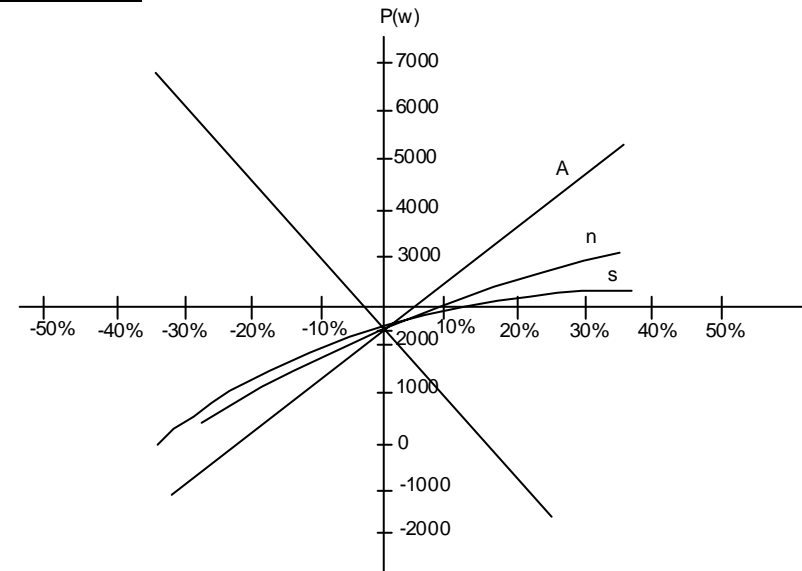
$$PW(10\%) = -11,500 + 3000[(p/A, 10\%, 6(1+n\%))] + [1000(p/F, 10\%, 6(1+n\%))]$$

When  $+n\%$  is  $40\%$

$$PW(10\%) = \dots\dots\dots$$

When  $-n\%$  is  $-40\%$

$$PW(10\%) \dots\dots\dots$$



Date: 2065/8/27

**8.1 Taxation law in Nepal:**

VAT = Value added tax = 13%

**8.2 Depreciation rate:**

**8.3 Recapture depreciation:**

**8.4 Taxes on normal gain.**

**8.5 Taxes on capital gain.**

**Chapter:9**

**9.0 Demand Analysis and sales forecasting:**

**Regression Analysis:**

This method involves.

- Determining the trend of Consumption by analysis past consumption statistics.
- Projecting future consumption by extrapolating trend.

Most commonly used relationship is the linear relationship.

$$y = a+bx$$

Where,  $y$  = Demand or sales for year  $t$ .

$x$  = Time variable.

$a$  and  $b$  = parameter.

$$b = (\sum xy - n\bar{x}\bar{y}) / (\sum x^2 - n\bar{x}^2)$$

**Example:**

Year (x)	Demand (y)	xy	$x^2$
1	13	13	1
2	14	28	4
3	17	51	9
4	18	72	16
5	18	90	25
6	19	114	36
7	20	140	49
8	22	207	81
9	23	220	100
10	22	264	125
11	24	288	144
12	24	325	169
13	25		
$\sum x = 91$	$\sum y = 259$	$\sum xy = 1998$	$\sum x^2 = 819$
$\sum x = 91$	$\sum y = 259$		
$\bar{X} = 7$	$\bar{y} = 19.92$		
$\sum xy = 1998$	$\sum x^2 = 819$		

$$b = (\sum xy - n\bar{x}\bar{y}) / (\sum x^2 - n\bar{x}^2)$$
$$= 1.018$$

$$a = \bar{y} - b\bar{x}$$
$$= 19.92 - 1.018 \times 7$$
$$= 12.794$$